Designing Formative Assessment Lessons in Mathematics

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Aiming to transform practice through design research:

- Analysing existing situations
- Designing new processes, products and experiences for teachers and learners
- Articulating values and principles that underpin these "designs"
- Analysing "designs in action"
- Revising and refining theories and designs in the light of these experiences
- "Scaling up" designs for use by others.

Meanwhile in England ...

Summative Assessment: GCSE Objectives

2015 Assessment Objectives		Weighting	
		Higher	Foundation
AO1	**Develop fluency and understanding** Use and apply standard techniques	40%	50%
AO2	**Reason and communicate** Reason, interpret and communicate mathematically	30%	25%
AO3	**Solve problems** Solve problems within mathematics and in other contexts	30%	25%

Modelling is specifically emphasized in GCSE

"Students should be aware that mathematics can be used to develop models of real situations and that these models may be more or less effective depending on how the situation has been simplified and the assumptions that have been made."

"Students can be said to have confidence and competence with mathematical content when they can apply it flexibly to solve problems."

Summative Assessment: GCSE Objectives

AO3		Weighting
	Formulate • translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes **Analyze and solve** • make and use connections between different parts of mathematics **Interpret** • interpret results in the context of the given problem **Evaluate** • evaluate methods used and results obtained • evaluate solutions to identify how they may have been affected by assumptions made.	30% (Higher) 25% (Foundation)

Modelling

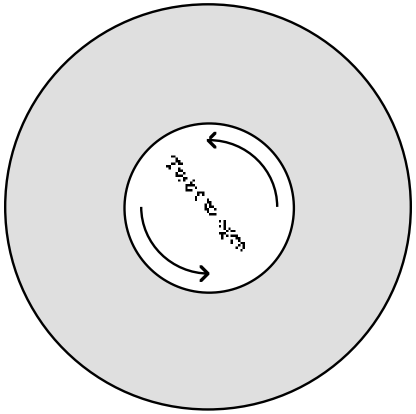
1. Last Sunday an accident caused a traffic jam 12 miles long on a two lane motorway. How many cars do you think were in the traffic jam?
   Explain your thinking and show all your calculations.
   Write down any assumptions you make.
   (Note: 5 miles is approximately equal to 8 kilometres)
2. When the accident was cleared, the cars drove away from the front, one car every two seconds. Estimate how long it took before the last car moved.

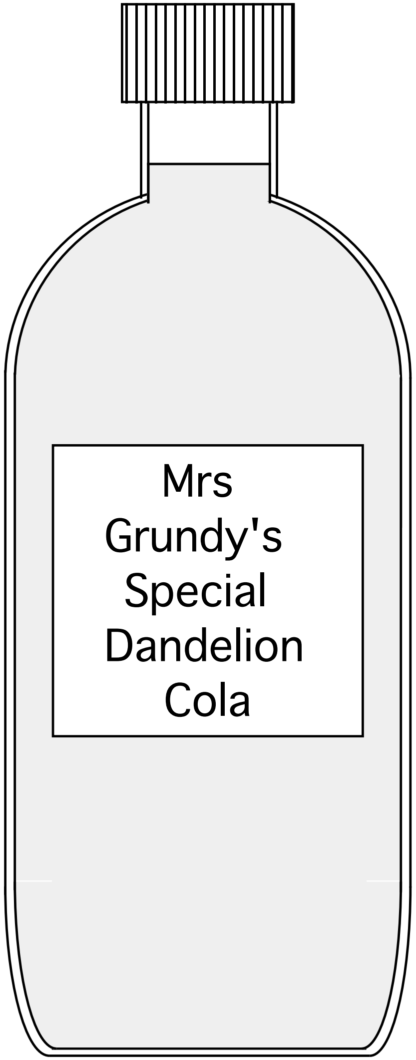
Modelling

Mrs. Grundy is planning to sell her home-made cola.
These pictures show the top and side views of the type of bottle she plans to use. They are drawn accurately, full size.

1. Calculate the volume of soda that is now in the bottle, in cubic centimetres.
   Do this as accurately as you can.
   Show your method clearly.
   State any formulae that you use.
2. Do you think that your calculation for the volume is too large or too small?
   Explain why you think this.

Top View





Side view

Now over to the USA .....

I’m calling on our nation’s governors and state education chiefs to develop **standards and assessments** that don’t simply measure whether students can fill in a bubble on a test, but whether they possess 21st Century skills like **problem solving** and **critical thinking** and **entrepreneurship** and **creativity**.

Remarks to the Hispanic Chamber of Commerce on a complete and competitive American Education March 10 2009.

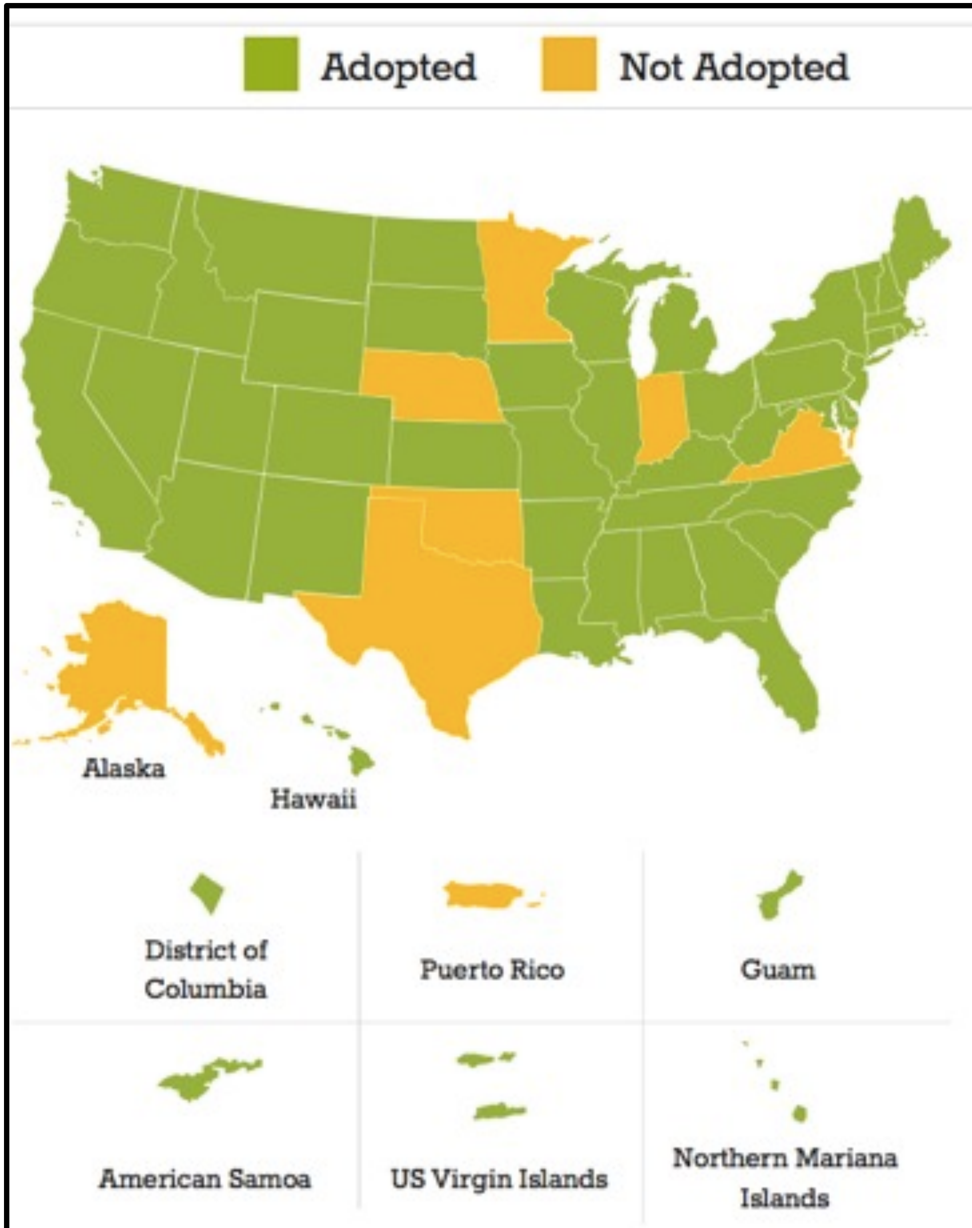
2010: The Common Core State Standards

- Prior to the CCSS, each US State had its own set of standards for Mathematics, K-12.
- Different states covered different topics at different grade levels.
- The CCSS were introduced to give consistency in learning materials and experiences across the nation.
- The National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) led their development.

“Teachers, parents, school administrators and experts from across the country together with state leaders provided input.”

- The federal government had no role in this development.

43 States have adopted the CCSS



COMMON CORE
STATE STANDARDS FOR

Mathematics

Mathematical Understanding

- *the ability to justify why a particular mathematical statement is true or where a mathematical rule comes from.*

Mathematical Practices

- *The ability to make strategic decisions when solving problems, to reason, to prove and communicate results*

Mathematical Practices (US)

- Make sense of complex problems and persevere in solving them.
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning.

(Source: Common Core State Standards for Mathematics)

From the Bill and Melinda Gates website:

BILL & MELINDA GATES foundation

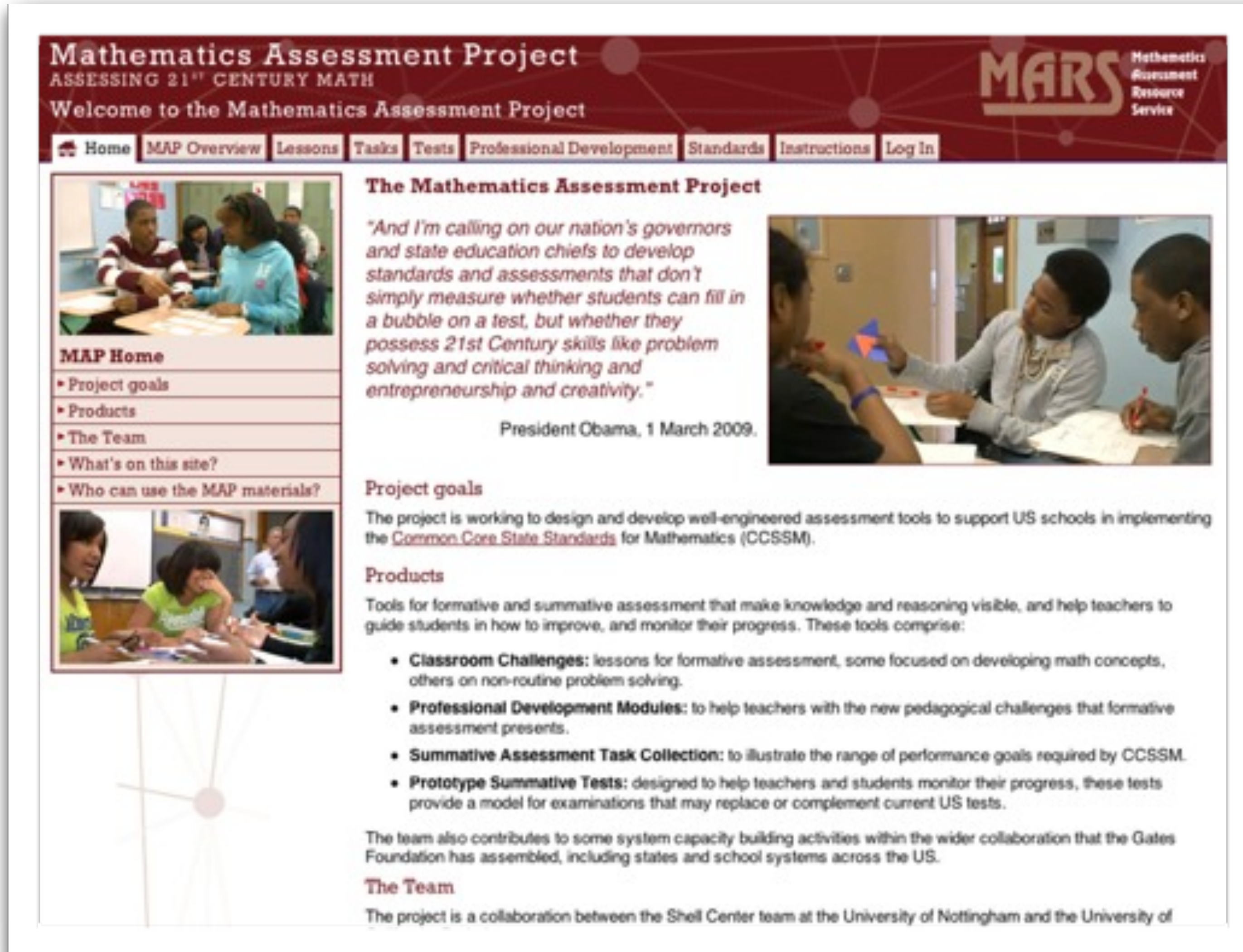
Only 25% of U.S. public high school graduates have the skills needed to succeed academically in college, which is an important gateway to economic opportunity in the United States.

Most of the country's K-12 public school teachers lack access to the tailored feedback, high-quality instructional materials, and support they need to do their best work and continually improve.

Together with our partners, we work to ensure that all students graduate from high school prepared to do college-level work.

Mathematics Assessment Project (MAP)

http://map.mathshell.org/materials/



What is a formative assessment lesson?

Formative assessment - Adaptive teaching

Students and teachers

Using evidence of learning

To adapt teaching and learning

To meet immediate needs

Minute-to-minute and day-by-day

(Thompson and Wiliam, 2007)

Findings from Black and Wiliam

- ***"All… studies show that… strengthening… formative assessment produces significant, and often substantial, learning gains. These studies range over ages, across several school subjects, and over several countries…"***
- ***Teachers emphasize grades - Students ignore comments when grades are also given.***
- ***"Feedback to any pupil should be about the particular qualities of his or her work, with advice on what he or she can do to improve, and should avoid comparisons with other pupils."***

Paul Black and Dylan Wiliam, "Assessment and Classroom Learning," *Assessment in Education*, March 1998, pp. 7-74.

Concept-focused Lessons

Reveal and develop students' interpretations of significant mathematical ideas and how these connect to their other knowledge.

- Number & Quantity
- Algebra
- Functions
- Modeling
- Statistics and Probability
- Geometry

Problem solving lessons

Reveal and develop students' capacity to apply their Math flexibly to non-routine, unstructured problems, both from pure math and from the real world.

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning.

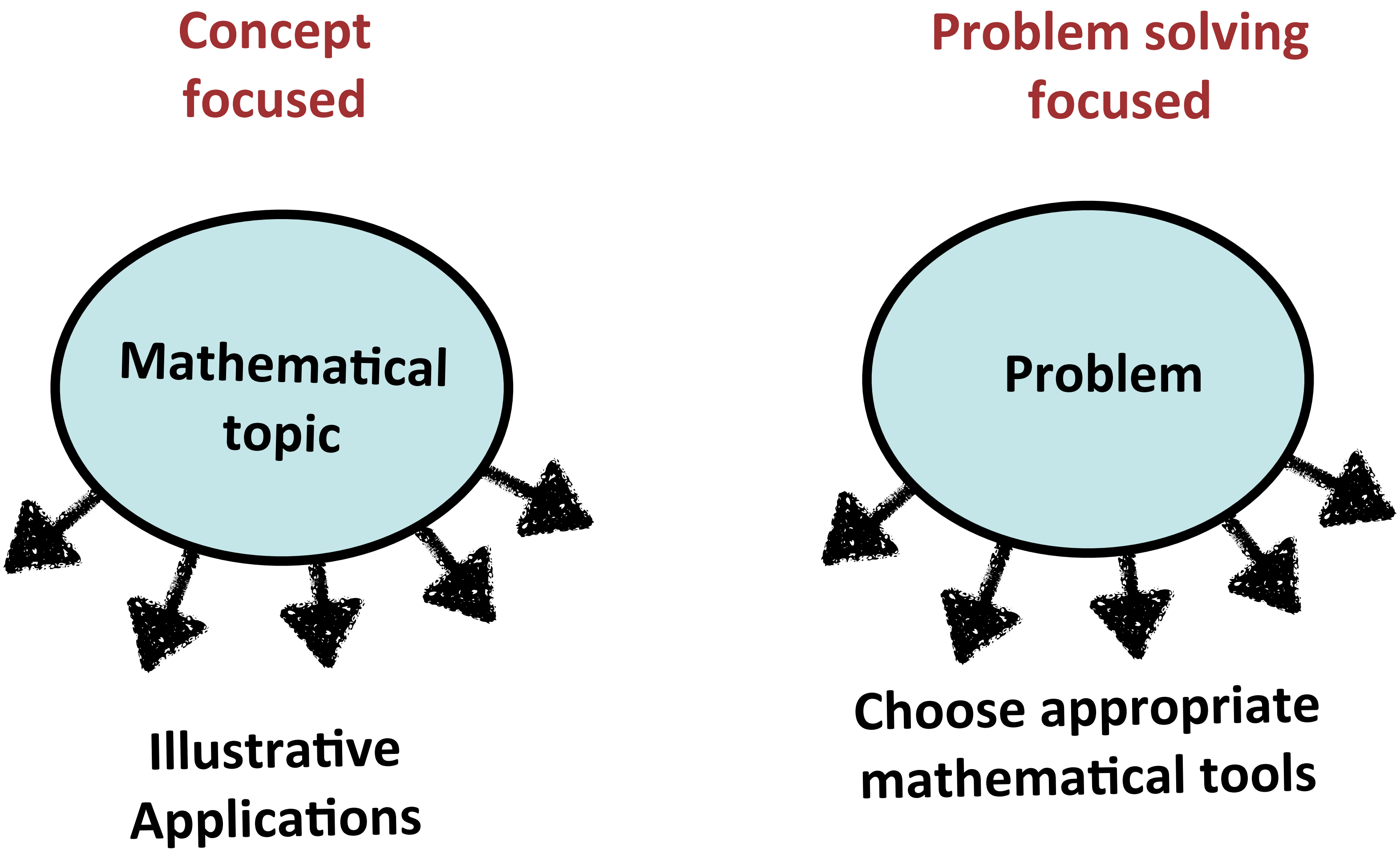
What is a problem?

" A problem is a task that the individual wants to achieve, and for which he or she does not have access to a straightforward means of solution."

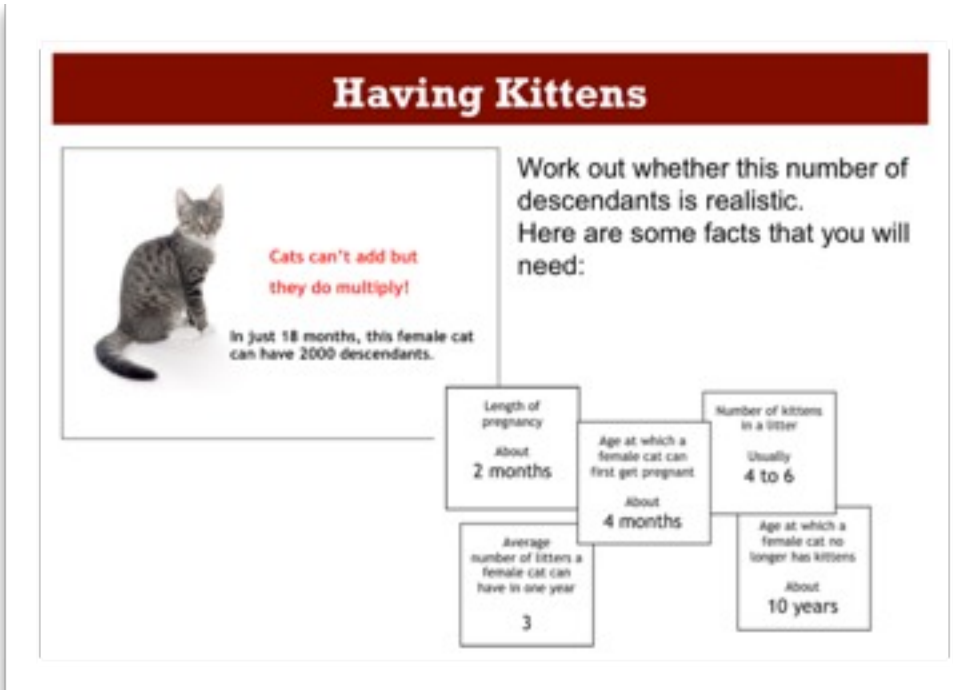
(Schoenfeld, 1985)

" .... problems should relate both to the application of mathematics to everyday situations within the pupils' experience, and also to situations which are unfamiliar. For many pupils this will require a great deal of discussion and oral work before even very simple problems can be tackled in written form."

(Cockcroft, 1982, para 249)


Concept focused
Problem solving focused
Mathematical topic
Problem
Illustrative Applications
Choose appropriate mathematical tools

100 sample lessons + PD Support
Grades 6, 7, 8 and High School

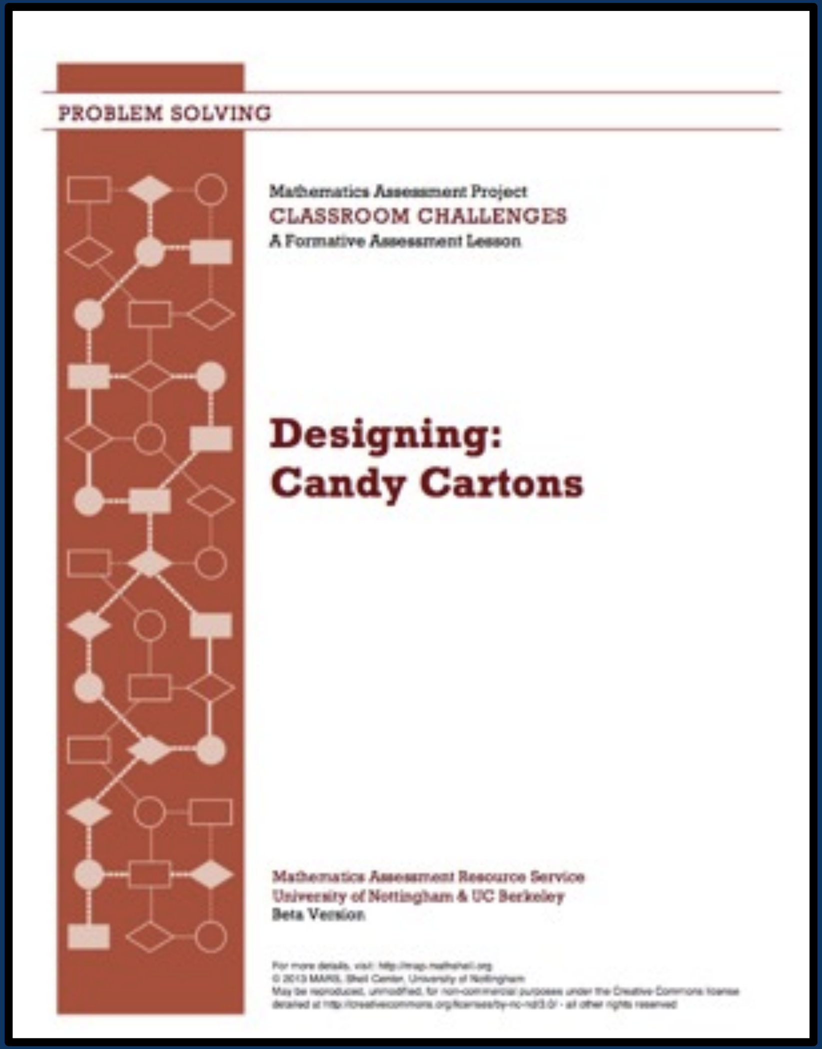


Think of each lesson plan as a research proposal.

MAP Professional Development Modules

- ▾ Supporting 21st Century Math Teaching
- ▸ 1: Formative Assessment
- ▸ 2: Concept Development Lessons
- ▸ 3: Problem Solving Lessons
- ▸ 4: Improving Learning Through Questioning
- ▸ 5: Students Working Collaboratively

Formative lessons in Problem Solving



What types of problems? What student roles?

Plan and organise

- Find an optimum solution subject to constraints.

Design and make

- Design an artifact or procedure and test it

Model and explain

- Invent, explain models, make reasoned estimates

Explore and discover

- Find relationships, make predictions

Interpret and translate

- Deduce information, translate representations

Evaluate and improve

- An argument, a plan, an artifact

Planning a problem solving lesson

Presentation (Hatsumon)

- Teacher presents problem in an intriguing way
- Students develop their ideas, individually

Developing a solution (Kikan-shido)

- Students share ideas
- Teacher observes students, selects student work

Comparing strategies (Neriage)

- Students share their solution ideas with whole class
- Students critique solutions, identifying strong and weak points.

Summarising and reflecting (Matome)

- Teacher summarises group findings, identifies important ideas, generalises
- Students summarise what they have learned themselves

Problem Solving Assessment Lesson

- **Initial, individual, unscaffolded problem**
  - Students tackle the problem unaided.
    Teacher assesses work and prepares qualitative feedback.
- **Individual work**
  - Students write responses to teacher's feedback
- **Collaborative work**
  - Students work together to produce and share joint solutions
- **Students compare different approaches using sample work**
  - Students discuss student work in small groups, then as a whole class
- **Whole class discussion: the payoff of mathematics**
  - Students improve their solutions to the initial problem,
    or one very much like it.
- **Individual reflection**
  - Students write about what they have learned.

Problem Solving Assessment Lesson

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Cats and Kittens

Cats can't add but they can multiply!

In just 18 months, this female cat can have 2000 descendants

Make sure your cat cannot have kittens

Is this figure of 2000 realistic ?

Number of kittens in a litter

Usually 4 to 6

Age at which a female cat gets pregnant

About 4 months.

Age at which a female cat no longer has kittens

About 10 years

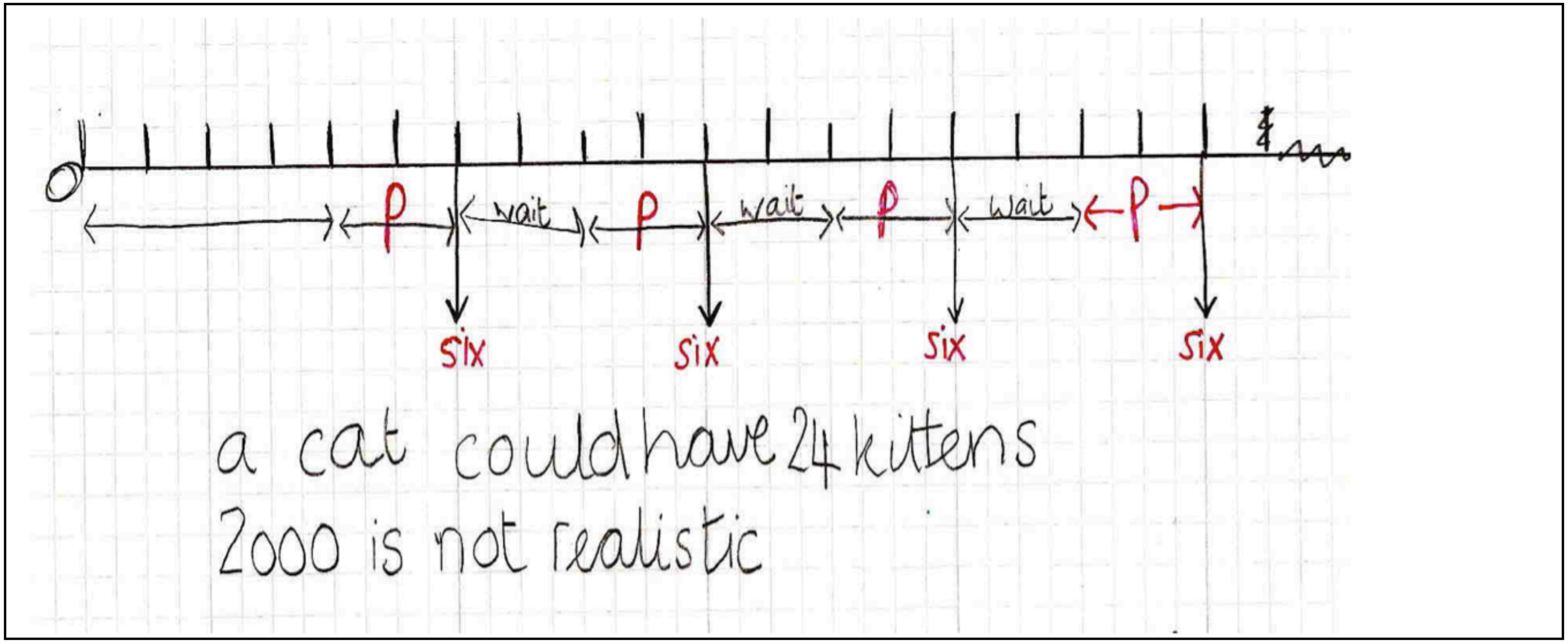
Average number of litters a female cat can have in one year

3

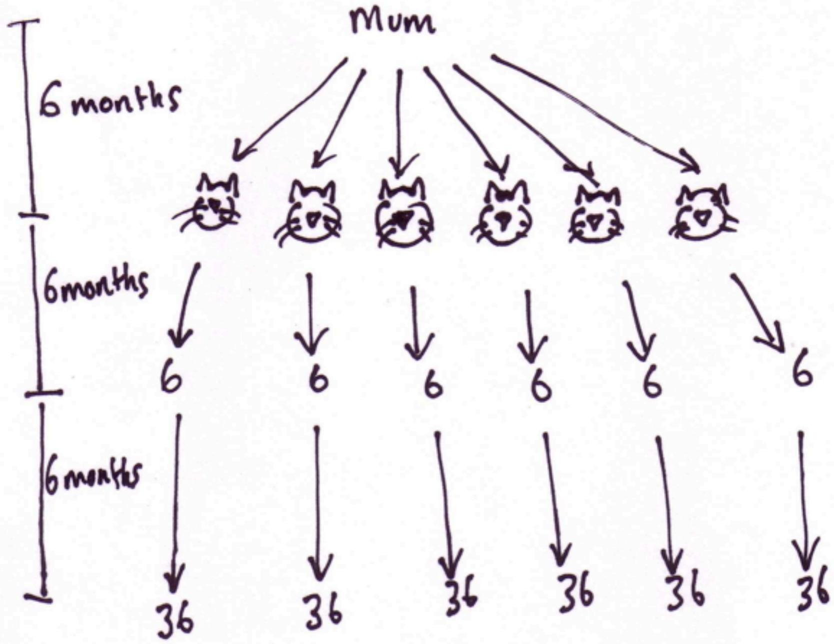
Length of pregnancy

About 2 months

Sample student work



Sample student work



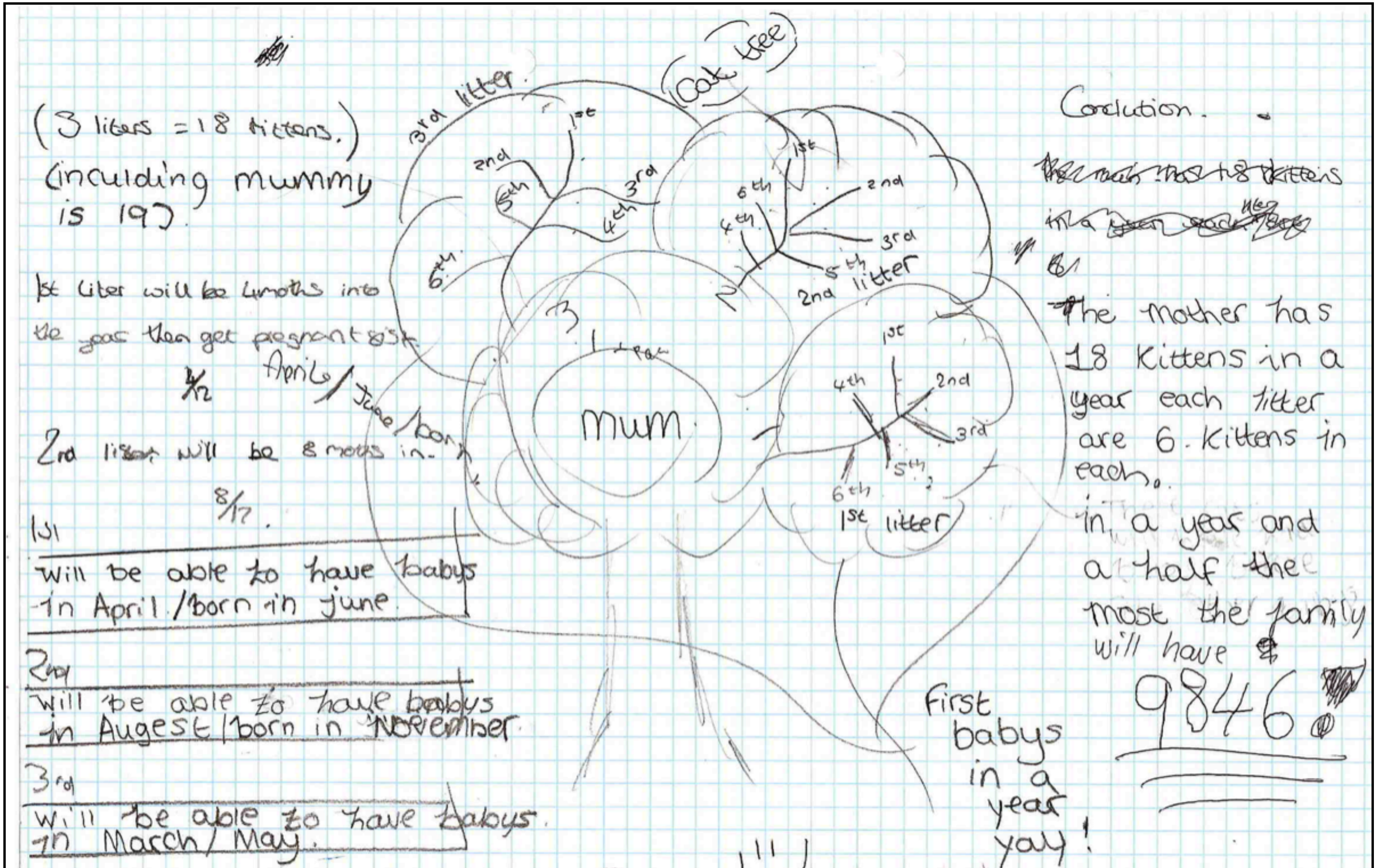
$$\text{Total cats} = 1 + 6\times 6 + 6\times 36$$

$$= 1 + 36 + 216$$

$$= \underline{\underline{253}}$$

So it's not realistic

Sample student work



Issue	Suggested questions and prompts
Has difficulty starting	• Can you describe what happens during first five months?
Does not develop suitable representation	• Can you make a diagram or table to show what is happening?
Work is unsystematic	• Could you start by just looking at the litters from the first cat? What would you do after that?
Develops a partial model	• Do you think the first litter of kittens will have time to grow and have litters of their own? What about their kittens?
Does not make clear or reasonable assumptions	• What assumptions have you made? Are all your kittens are born at the beginning of the year?
Makes a successful attempt	• How could you check this using a different method?

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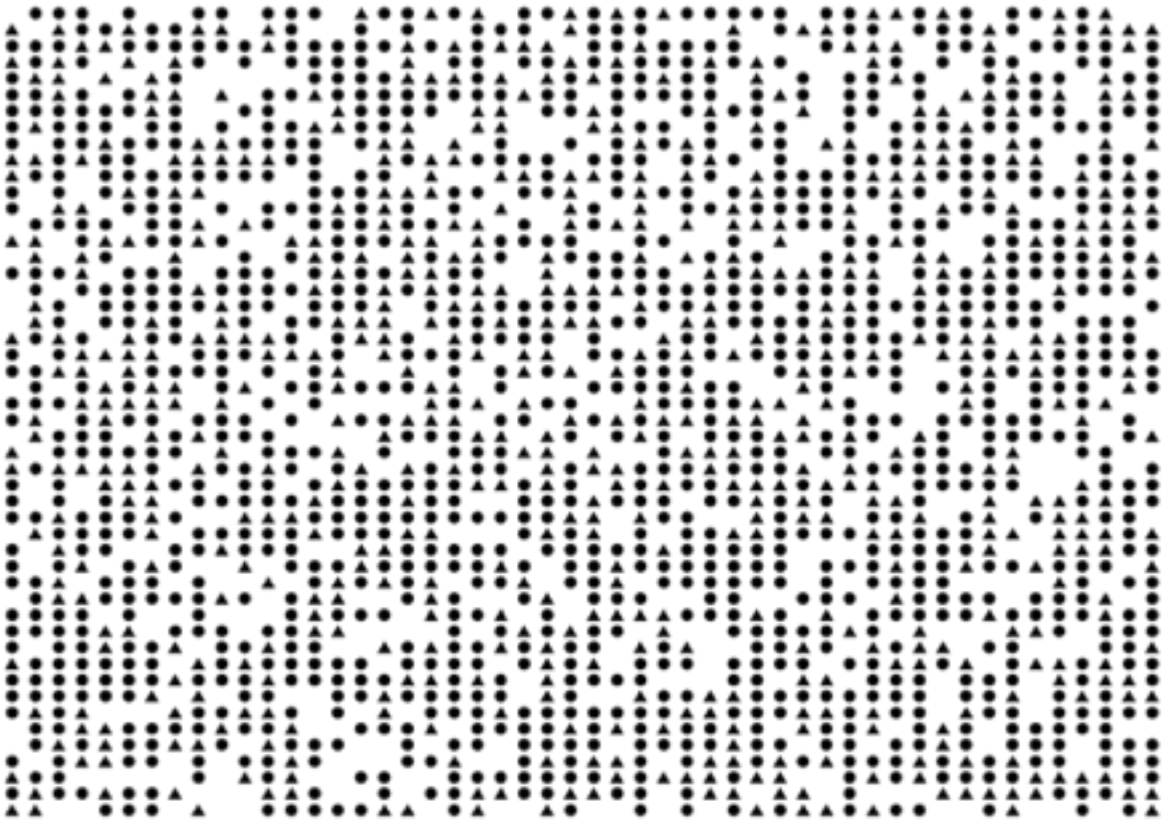
    or one very much like it.
- **Individual reflection**
  - Students write about what they have learned.

	Representing	Analysing	Interpreting and evaluating	Communicating and reflecting
	Draws a simple diagram or Draws a timeline with some key events shown sequentially	Finds the number of kittens that would exist if each cat had only one litter	Relates their findings to the original problem, e.g. by stating whether 2000 descendants is or is not realistic	Presents work in such a way that it is possible to determine which is the original cat, and how many kittens are within each litter
PROGRESSION	Draws a simple diagram and shows or implies multiplication is an appropriate mathematical tool *or* Draws a timeline with some key events shown sequentially, considering more than just the offspring of the first cat	Uses multiplication to find the number of kittens that would exist if each cat had only one litter and recognises the need to count all those descendants	Makes explicit the assumption about the number of kittens per litter, e.g. 'Each litter is 6 kittens'	Shows methods so that someone else can follow their reasoning reasonably well
	The chosen method represents both multiplication and time for the original kitten even if not all her descendants are represented	Recognises that most cats, in the time available, can have more than one litter	Qualifies assumptions about the number of kittens per litter. E.g. 'I used 6 – that gives the biggest number of cats'	Throughout the task there is clear, effective and concise communication that builds to a solution, even if partial

PROGRESSION	diagram and shows or implies multiplication is an appropriate mathematical tool *or* Draws a timeline with some key events shown sequentially, considering more than just the offspring of the first cat	to find the number of kittens that would exist if each cat had only one litter and recognises the need to count all those descendants	assumption about the number of kittens per litter, e.g. 'Each litter is 6 kittens'	that someone else can follow their reasoning reasonably well
	The chosen method represents both multiplication and time for the original kitten even if not all her descendants are represented	Recognises that most cats, in the time available, can have more than one litter	Qualifies assumptions about the number of kittens per litter. E.g. 'I used 6 – that gives the biggest number of cats'	Throughout the task there is clear, effective and concise communication that builds to a solution, even if partial
	The chosen method represents both multiplication and time for the original kitten and all her descendants	Uses an effective method to work towards a credible solution that takes into account the wide range of factors within the task	Makes explicit further assumptions. E.g. No cats die or that cats become pregnant as soon as physically possible	Throughout the task there is clear, effective and concise communication with evidence of reflection. E.g. The number of kittens per litter affects the outcome significantly

Counting Trees



● The circles show **old** trees

▲ The triangles show **young** trees

- Think of a method you could use to estimate the number of trees of each type.
- Explain the method fully
- Use your method to estimate the number of old trees and young trees

Issue	Suggested questions and prompts
Method doesn't use sampling E.g. Multiplies number of rows by number of columns.	• What assumptions have you made?
Sample chosen is unrepresentative E.g. Counts trees in first row, then multiplies by number of rows.	• How could you improve your estimate? • Is your sample size reasonable? • Which rows or columns have you considered?
Student uses area and/ or perimeter	• What assumptions have you made?
Makes incorrect assumptions E.g. Does not account for gaps. E.g. Assumes equal amounts of each type.	• Does your work assume that there is a pattern to how the trees are distributed?
Reasoning is difficult to follow	• Would someone unfamiliar with the task understand your work?
Appropriate method chosen	• How could you check your result? Can you find a different sampling method?

A pedagogical Problem

Students may not create their own powerful approaches.

If we tell them to try a particular approach, opportunities for decision-making are taken away from the student. The problem solving lesson may even become an ***exercise*** in imitating our method - a method that carries authority.

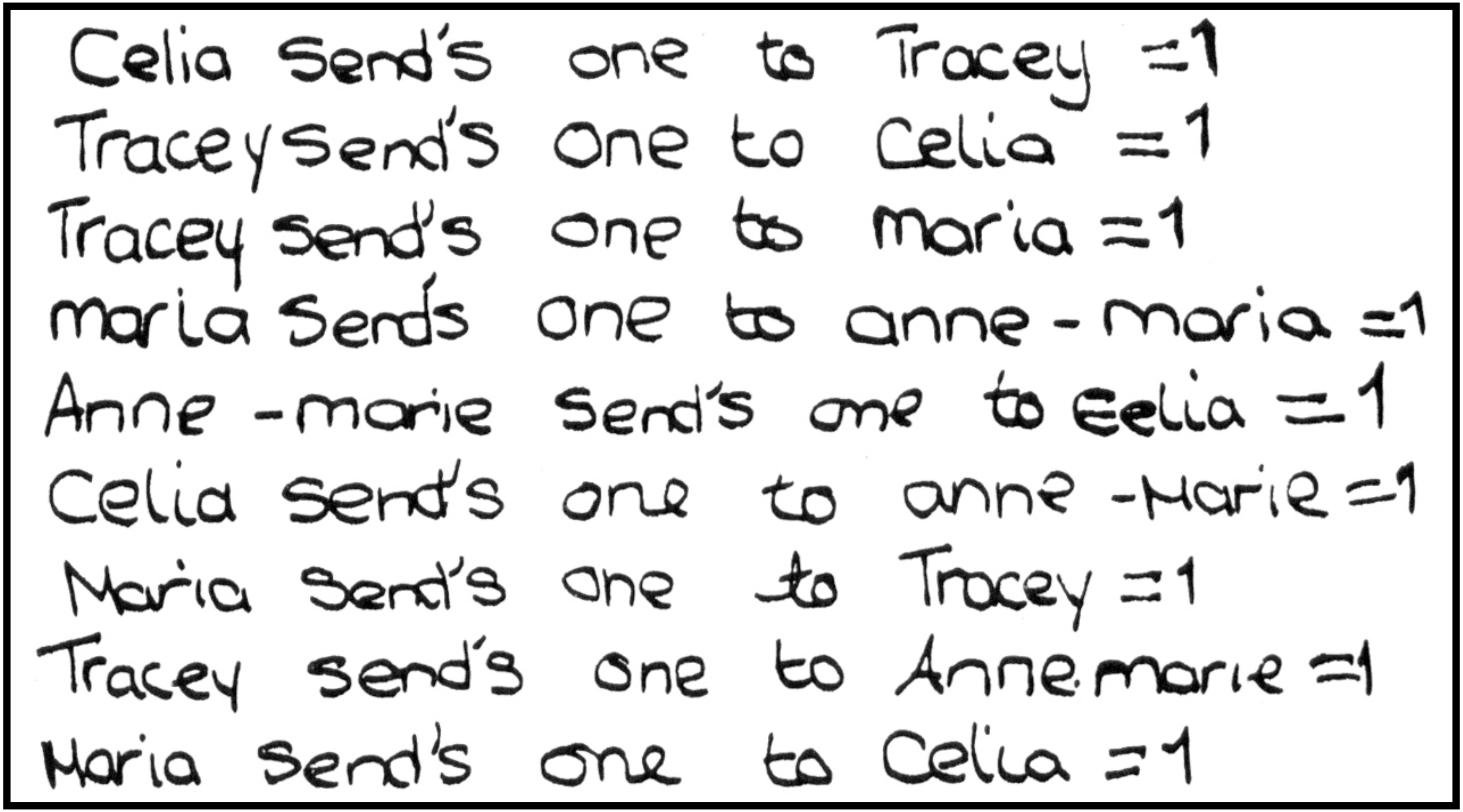
So how can we introduce more powerful approaches, without also removing student decision-making?

Model and Explain

Text Messaging

- How many text messages are sent if four people all send messages to each other?
- How many text messages are sent with different numbers of people?
- Approximately how many text messages would travel in cyberspace if everyone in your school took part?
- Can you think of other situations that would give rise to the same mathematical relationship?

We offer students sample work to critique...


Celia Send's one to Tracey =1
Tracey Send's One to Celia =1
Tracey send's one to Maria =1
maria Sends one to anne - maria =1
Anne -marie send's one to Celia =1
Celia send's one to anne -Marie =1
Maria Send's one to Tracey =1
Tracey send's one to Anne marie =1
Maria Send's one to Celia =1

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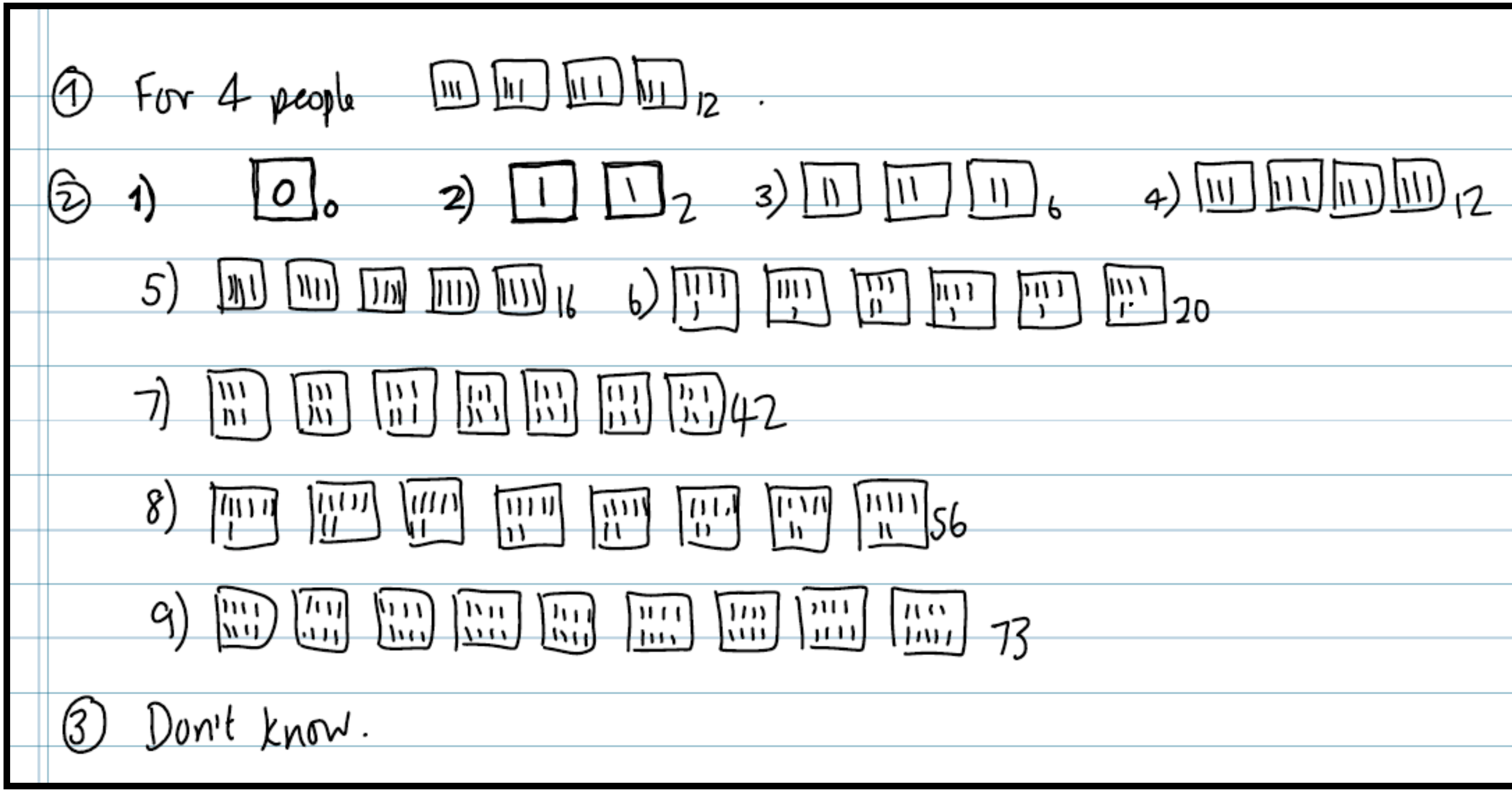


We offer students sample work to critique...

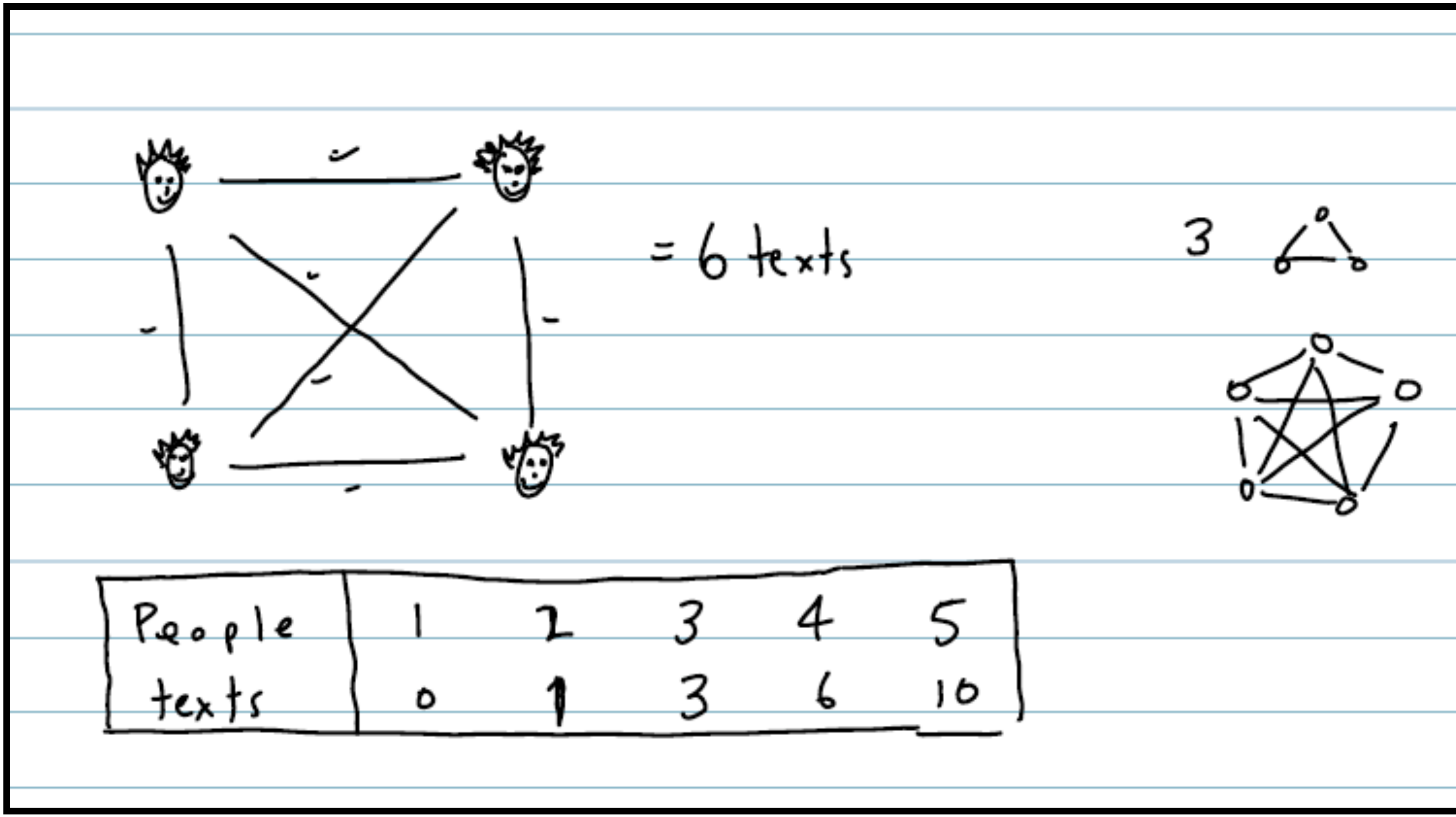
	Amy	Belinda	Suzie	Mary	Tom
Amy	—	Text	Text	Text	Text
Belinda	Text	—	Text	Text	Text
Suzie	Text	Text	—	Text	Text
Mary	Text	Text	Text	—	Text
Tom	Text	Text	Text	Text	—

= 12 texts for 4 people

Tom adds 8 more texts = 20 altogether.

For more people you add extra rows and colums.

We offer students sample work to critique...



Some possible uses of “sample student work”

- To help students that are making little progress with a problem or who have become fixated on a single line of enquiry
- To encourage metacognitive behaviour: stepping back from ‘working through’, to ‘reflecting on’ advantages and disadvantages of alternative approaches

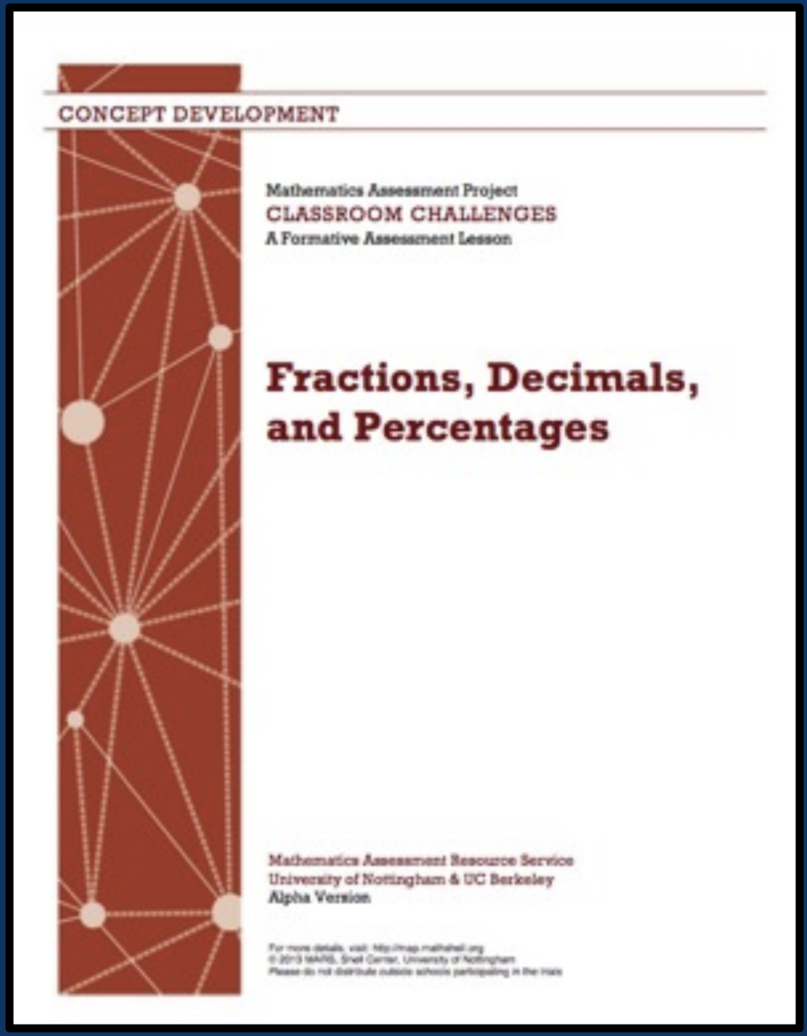
- To encourage students to make connections within mathematics
- To draw attention to common errors and misconceptions

- To encourage criticality without fear of criticism
- To become more aware of valued criteria for assessment, e.g. students assess the work using criteria

Where is the assessment in all this?

- Teachers gather information on what students can do unaided;
- Teachers listen and monitor students while they work, and offer support through questioning, as this is needed;
- Students gain constructive feedback via other students, and the teacher, as student work is discussed and developed;
- Students act on feedback by improving and refining their responses;
- Teachers get feedback on learning by observing the development of student work through successive revisions.

Formative lessons for developing conceptual understanding



Principles derived from empirical studies

Expose existing ideas and concepts

- ‘pull back the rug’

Confront with implications and contradictions

- provoke ‘tension’ and ‘cognitive conflict’

Resolve conflict through discussion

- allow time for the formulation of new concepts and methods.

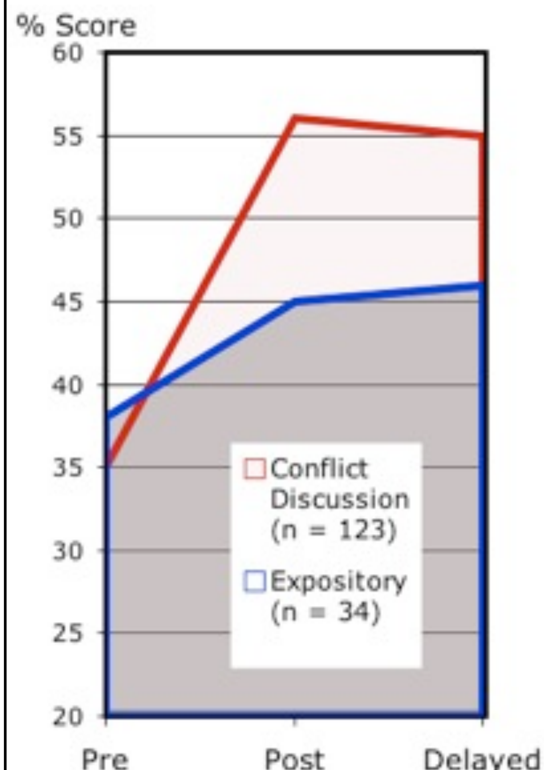
Generalize, extend and link learning

- applying the new concepts and methods on further problems.

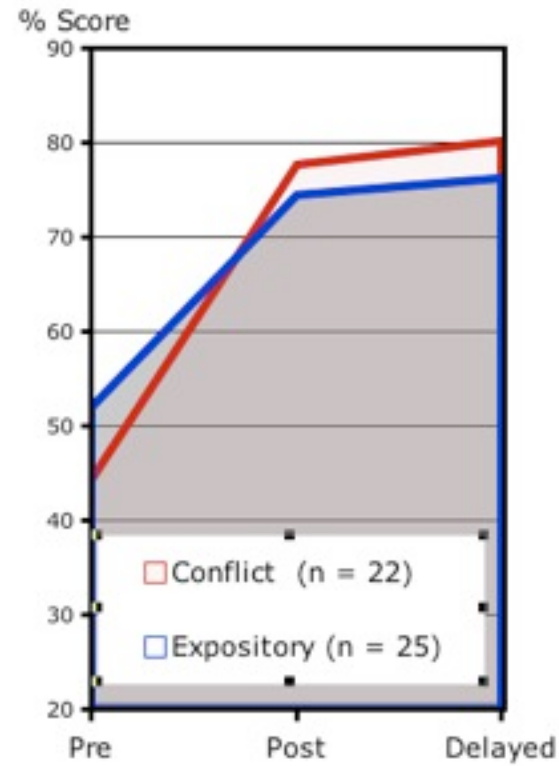
Formative assessment teaching compared with...

Guided discovery “Reflections”	Expository “Rates”	Formative + expository “Decimals”

Birks (1987)



Onslow (1986)



Swan (1983)

Concept Assessment Lesson

- **Initial, individual task**
  - An assessment task is presented and a range of responses are evoked. The task is put to one side.
- **Collaborative discussion task**
  - Prior conceptions are discussed and debated.
    Teacher aims to provoke cognitive conflict through questioning.
- **Whole class discussion**
  - Pre-conceptions are explicitly challenged.
  - Sample student work illustrating ‘misconceptions’ may be used.
- **Revisit initial task or a similar one**
  - The assessment task is re-examined and responses are improved. Students describe what they have learned.

Understanding

Mental operations involved in understanding:

- **Identification**: we can bring the concept to the foreground of attention, name and describe it.
- **Discrimination**: we can see similarities and differences between this concept and others.
- **Generalization**: we can see general properties of the concept in particular cases of it.
- **Synthesis**: we can perceive a unifying principle

(Sierpinska 1994)

Task "genres" that generate discussion

Classifying, naming and defining objects

- what is the same and what is different?

Interpreting multiple representations

- what is another way of showing this?

Analyzing and testing generalizations

- "always, sometimes or never true?"

Exploring structure and connections

- what happens if I change this?
- How will it affect this?

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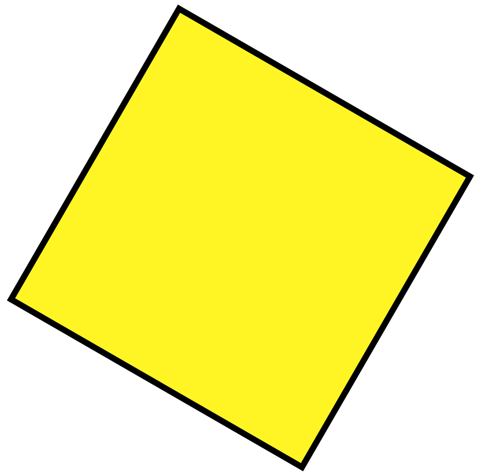
Classifying and defining

Four equal sides

Diagonals meet at right angles



Two pairs of parallel sides

4 lines of symmetry

Two equal diagonals

Rotational symmetry of order 4

Four right angles

A1 The diagonals of the shape are congruent	A2 The shape has at least one side that is 5cm long	A3 The diagonals of the shape bisect each other at right angles	A4 The shape has 4 equal angles	A5 The shape has two pairs of parallel sides

B1 The shape has at least one side that is 4cm long	B2 The diagonals of the shape bisect each other	B3 The shape has 4 equal angles	B4 Opposite sides of the shape are congruent	B5 The shape has at least one side that is 6cm long

C1 The diagonals of the shape are not congruent	C2 The shape has at least one side that is 12cm long	C3 The shape has at least one side that is 7cm long	C4 The shape contains at least one 55° angle	C5 Opposite sides of the shape are parallel

D1 The diagonals of the shape bisect each other at right angles	D2 All four sides are congruent	D3 The shape contains at least one 70° angle	D4 Opposite sides of the shape are parallel	D5 The shape has at least one side that is 7cm long

Classifying and defining

Quadratic function

Maximum value of 9

$$y = -(x-4)(x+2)$$

Crosses x axis at (4,0)

Factorizes with integers

Line of symmetry $x = 1$

Crosses y axis at (0,8)

Which *pairs* of statements *define* the quadratic?

Classifying and defining

The graph of y against x is a straight line

$y \div x$ always gives the same result

If $x = 0$, then $y = 0$

$y = kx$ where k is a constant.

y is proportional to x

When x doubles in value, y doubles in value.

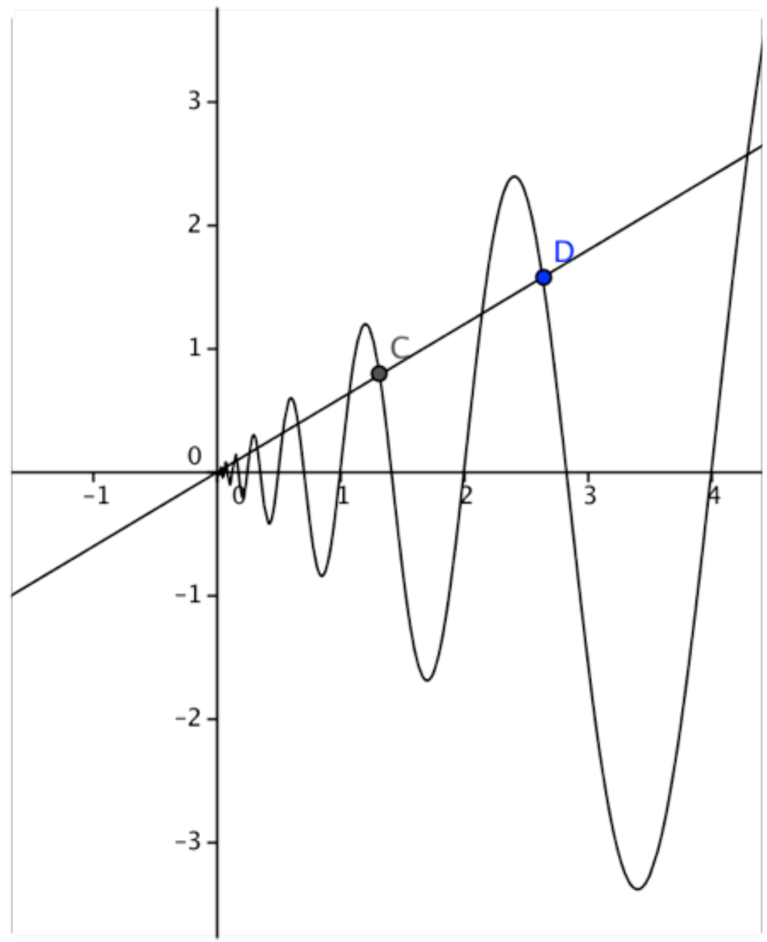
When x increases by equal steps, y also increases by equal steps.

Which statements *define* proportion?

Classifying and defining: counterexample.

When x doubles in value, y doubles in value.

$$y = x \sin\left(2\pi \frac{\ln x}{\ln 2}\right)$$



Proportion or non-proportion?

CYCLE

It takes .... minutes to cycle .... miles.

PETROL

...... litres cost £ ......

MONEY

£ ...... is worth the same as ...... dollars

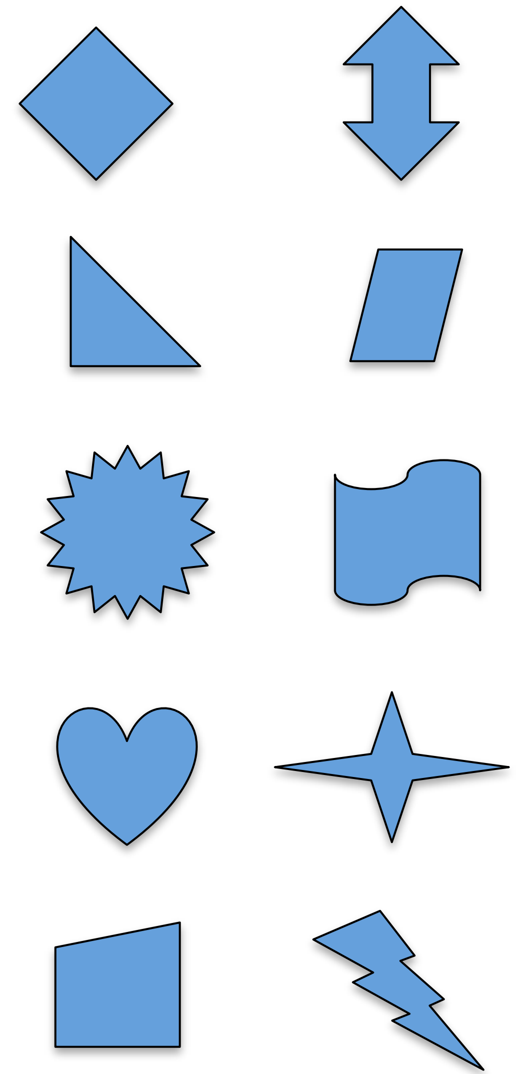
DRIVING

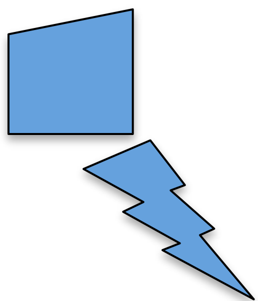
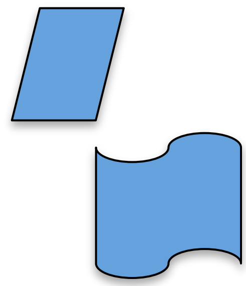
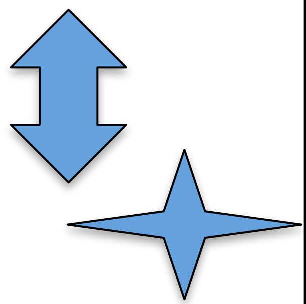
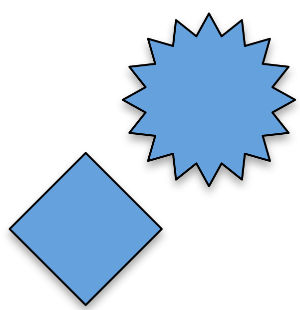
If I drive at .... miles per hour, the journey will take .... hours.

FIRE

It would take ...... minutes to vacate a building if we put in ...... fire escapes.

	No rotational symmetry	Rotational symmetry
No lines of symmetry		
One or two lines of symmetry		
More than two lines of symmetry		



	No rotational symmetry	Rotational symmetry
No lines of symmetry		
One or two lines of symmetry		
More than two lines of symmetry		

Is it possible to find a shape that has no rotational symmetry which has more than two lines of symmetry?

$y = x^2 + 2x + 4$

$y = 4x^2 - 4x + 1$

$y = x^2 - 5x + 4$

$y = x^2 - 4x + 4$

$y = x^2 + 7x - 3$

$y = 2x^2 - 5x - 3$

	Factorizes with integers	**Does not factorize with integers**
Two *x* intercepts		
One *x* intercept		
No *x* intercepts		

Is it possible to find a quadratic function *y*=f(*x*) that factorizes but has no *x* intercepts?

	Factorizes with integers	Does not factorize with integers
Two *x* intercepts	$y = x^2 - 5x + 4$	$y = 2x^2 - 5x - 3$ $y = x^2 + 7x - 3$
One *x* intercept	$y = x^2 - 4x + 4$	$y = 4x^2 - 4x + 1$
No *x* intercepts		$y = x^2 + 2x + 4$

Task “genres” that generate discussion

Classifying, naming and defining objects

- what is the same and what is different?

Interpreting multiple representations

- what is another way of showing this?

Analyzing and testing generalizations

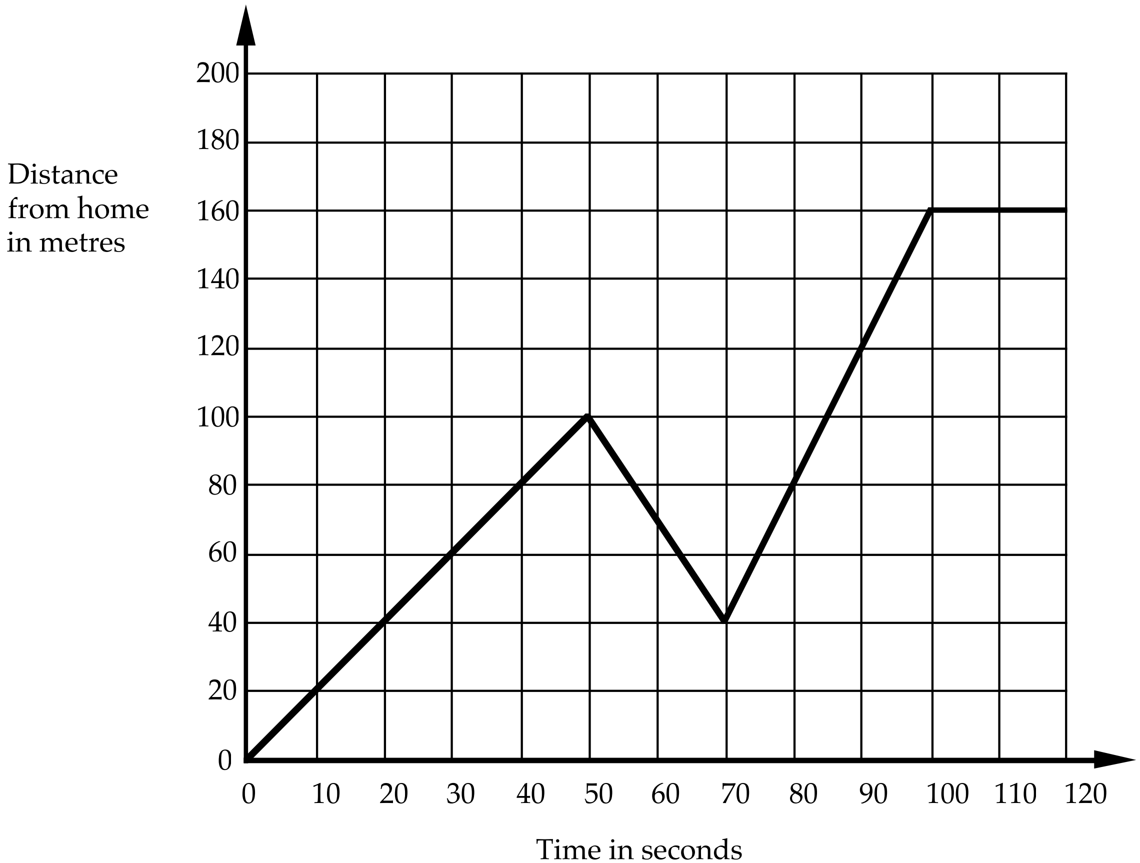
- “always, sometimes or never true?”

Exploring structure and connections

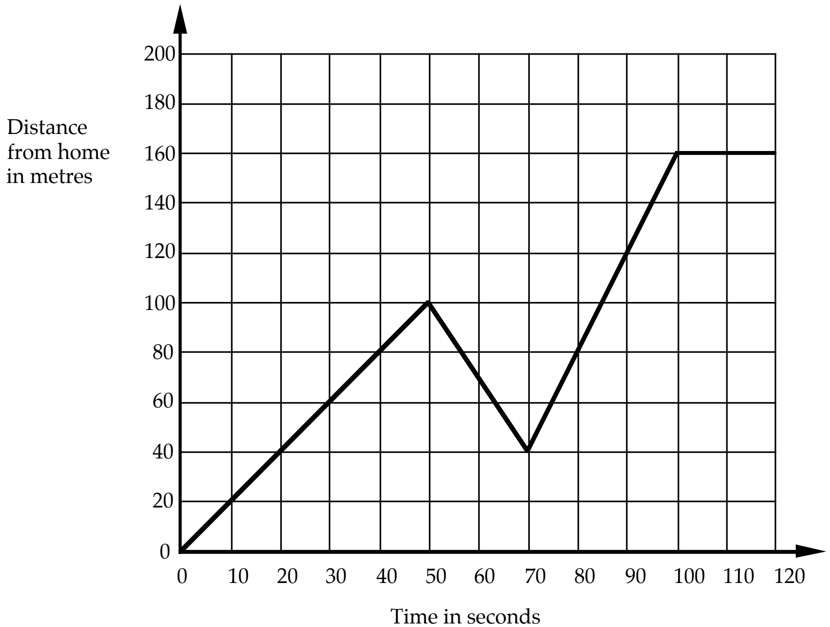
- what happens if I change this?
- How will it affect this?

Multiple representations: Distance-time graphs

Every morning Jane walks along a straight road to a bus stop 160 metres from her home, where she catches a bus to college.
The graph shows her journey on one particular day. Describe what may have happened. Is the graph realistic? Why?



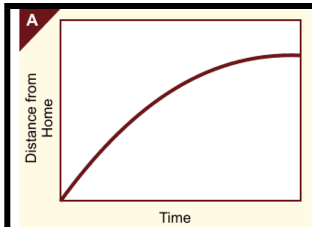
Assessment Task: Some Common Issues

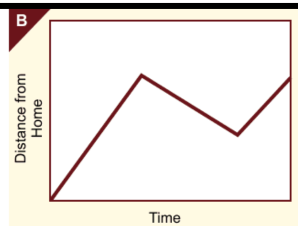


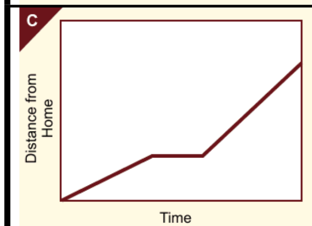
Tom walked along a road for 100metres instead of walking another 30metres he took a short cut down an alleyway which took he 20 minutes the walked very quickly then he caught the bus to his college which took about 50minutes.

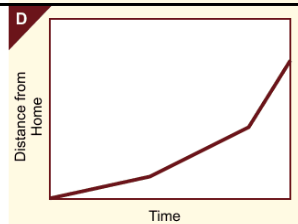
when he get out he starts walking fast to the bus stop then he slows down the he picks up the speed again and then ~~the~~ this speed goes ~~cons~~ constant.

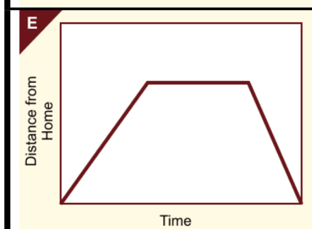
Matching graphs and stories

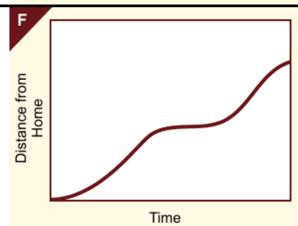


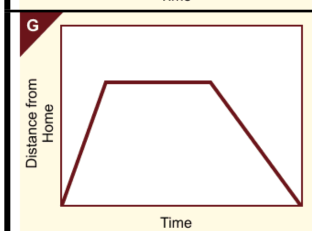


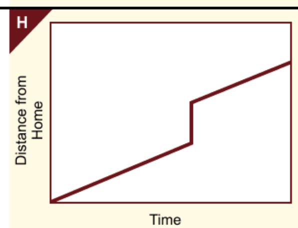


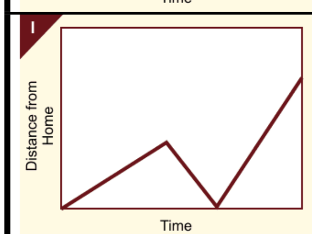


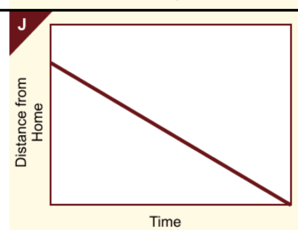












1 Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.

2 Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.

3 Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.

4 Tom walked slowly along the road, stopped to look at his watch, realized he was late, and then started running.

5 Tom left his home for a run, but he was unfit and gradually came to a stop!

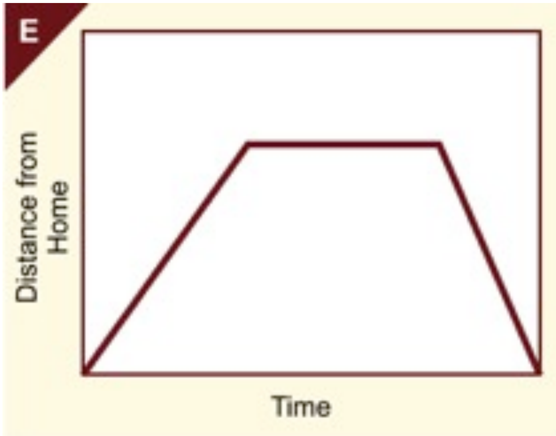
6 Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.

7 Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.

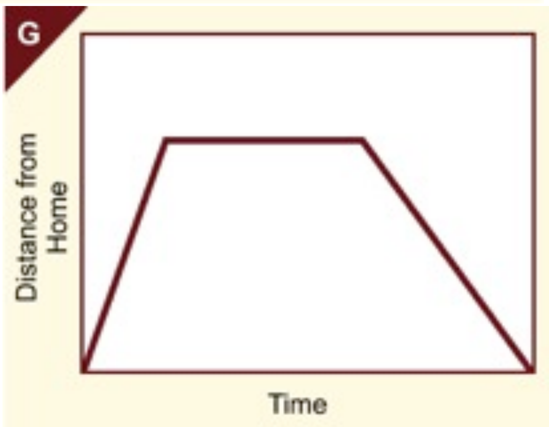
8 This graph is just plain wrong. How can Tom be in two places at once?

9 After the party, Tom walked slowly all the way home.

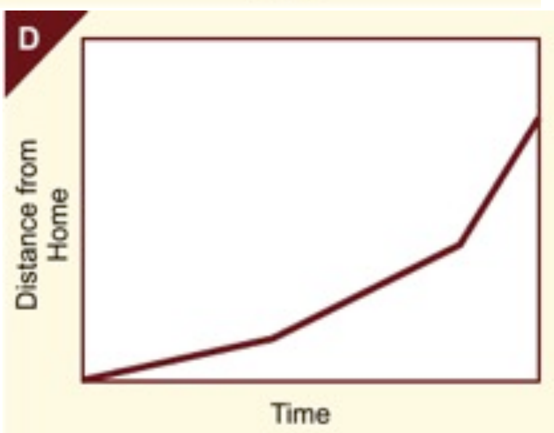
10 Make up your own story!



2 Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.



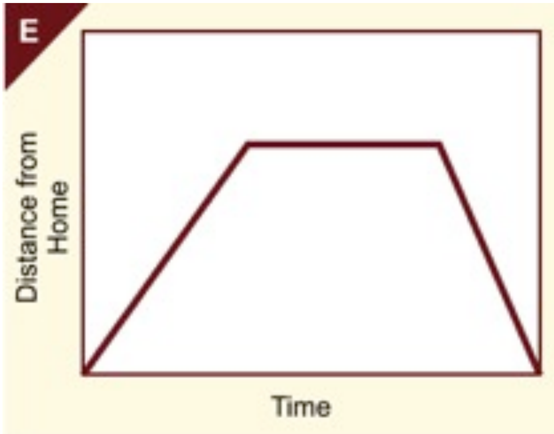
1 Tom ran from his home to the bus stop and waited.
He realized that he had missed the bus so he walked home.



6 Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.

Ambiguity promotes discussion.

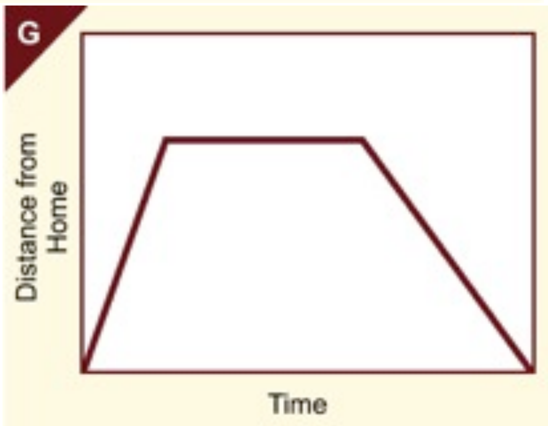
E.g. Can the distance from home be constant, yet Tom still be moving?



2 Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.

Q

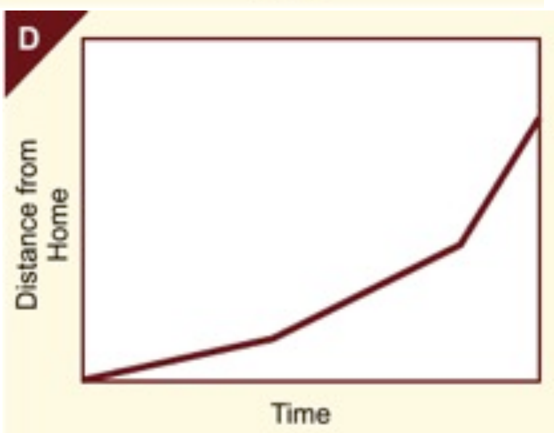
Time	Distance
0	0
1	10
2	20
3	40
4	60
5	120



1 Tom ran from his home to the bus stop and waited.
He realized that he had missed the bus so he walked home.

P

Time	Distance
0	0
1	40
2	40
3	40
4	20
5	0



6 Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.

T

Time	Distance
0	0
1	20
2	40
3	40
4	40
5	0

Whole-Class Discussion & Review of Work

Extending to new examples

- Show me an example of a graph that represents this story....
  "Sam ran out of his front door, then slipped and fell. He got up and walked the rest of the way to school."
- Show me an example of a story that fits this graph...
- Show me an example of a table that fits this graph...

Generalising principles and methods

- How can you tell from a graph if a person is running, walking?
- How can you tell this from a table?
- Why does your method work?

Linking to other ideas...

- What would be the speed if the equation of the graph was....
- How does this work relate to what you did in science?

Students are given the opportunities to improve their responses to the initial assessment task.

Formative Assessment with Computer Technologies

- Students work on a large, scroll-able and zoom-able canvas.
- They share screens and use finger gestures to navigate.
- Use stylus to write, move and change information.
- Their reasoning may be shared between tablets and on the screen at the front of the class.
- The tablet can perhaps give students intelligent feedback as they work in the form of formative questions.

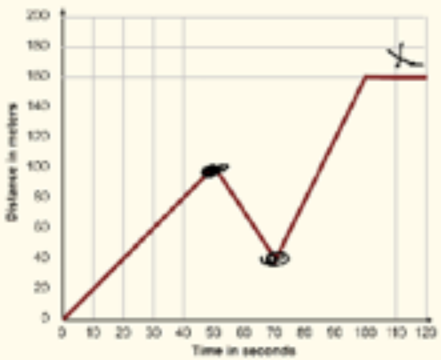
FactArt

RESET UNDO REDO PEN ERASER FILE ADD CARD TEACHERS

Journey to the Bus Stop

Every morning, Tom walks along a straight road to a bus stop, a distance of 160 meters.

The graph shows his journey on one particular day.



x-this woold Be where he was waiting for the Bus!

- Stopped
- Stopped again

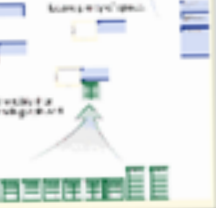
Describe what may have happened. Include details like how far he walked.

he could have missed the bus. Because he met one of his friends and then ran to the busstop

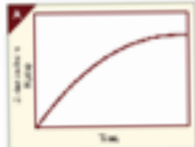
Are all stages of the graph realistic? Fully explain your answer.

NO!
- he coouldn't Stop.
- he would have kept walking.
-

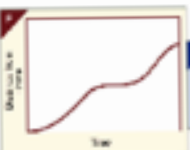
19:56



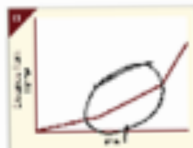
the graph it could
show the time that
he took to look at
his watch



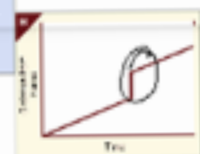
These 2 match because the story matches the graph as if the story was the graph.



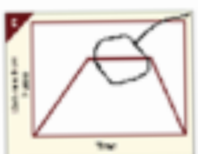
These graphs looks like they Match because it shows he sped up and slowed down Also the metres got longer where he slowed down.



Showed shows that Tom gradually came to a stop



How could Tom go say 20 metres in the same amount of time it took him to get say 100 metres.



The middle part of the graph could suggest that he is waiting for the bus to arrive and the last part is him walking home.

Multiple representations: Functions & Situations

E. Cooling kettle

A kettle of boiling water cools in a warm kitchen.

x = the time that has elapsed in minutes.

y = the temperature of the kettle in degrees Celsius.

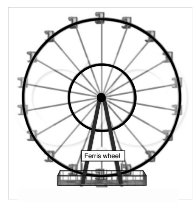
What is the temperature of the room?

F. Ferris wheel

A Ferris wheel turns round and round.

x = the time that has elapsed in seconds.

y = the height of a seat from the ground in meters.



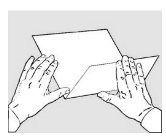
How long does it take the Ferris wheel to turn once?

G. Folding paper

A piece of paper is folded in half. It is then folded in half again, and again…

x = the number of folds.

y = the thickness of the paper in inches.



How thick would the paper be if you could fold it 10 times?

H. Speed of golf shot

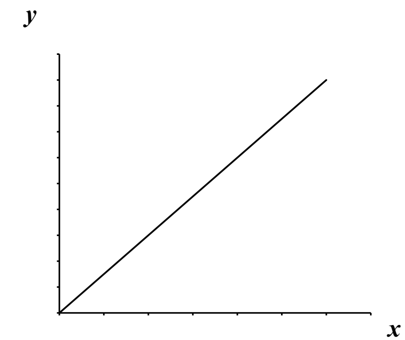
A golfer hits a ball.

x = the time that has elapsed in seconds.

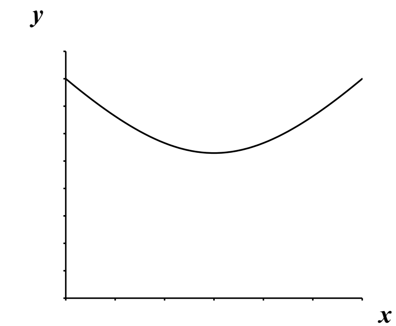
y = the speed of the ball in meters per second.

When is the ball travelling most slowly?

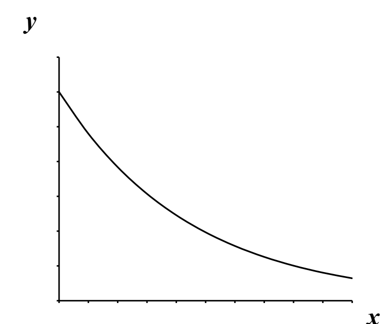
G7



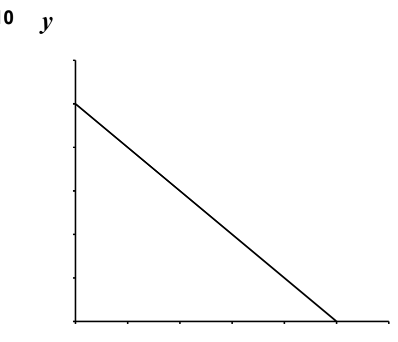
G8



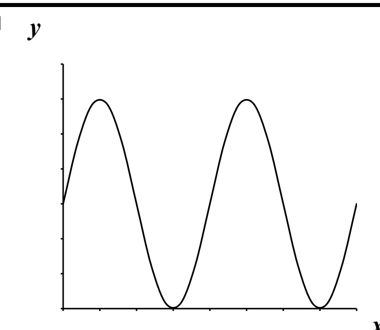
G9



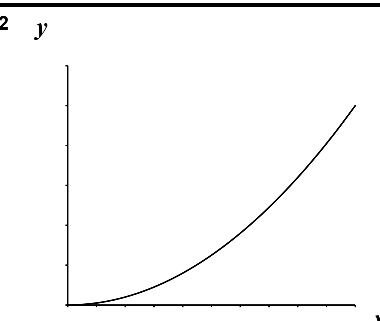
G10



G11



G12



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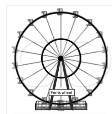

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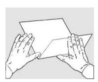

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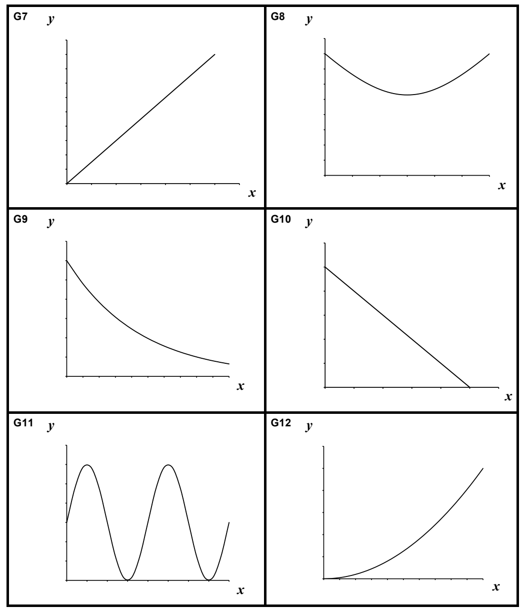
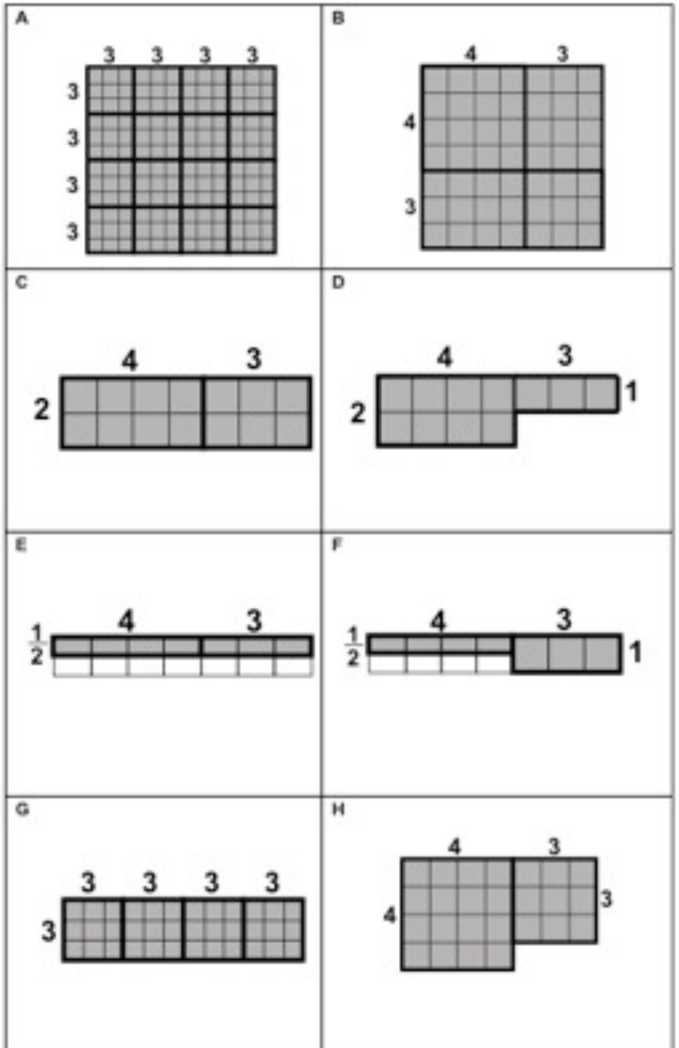


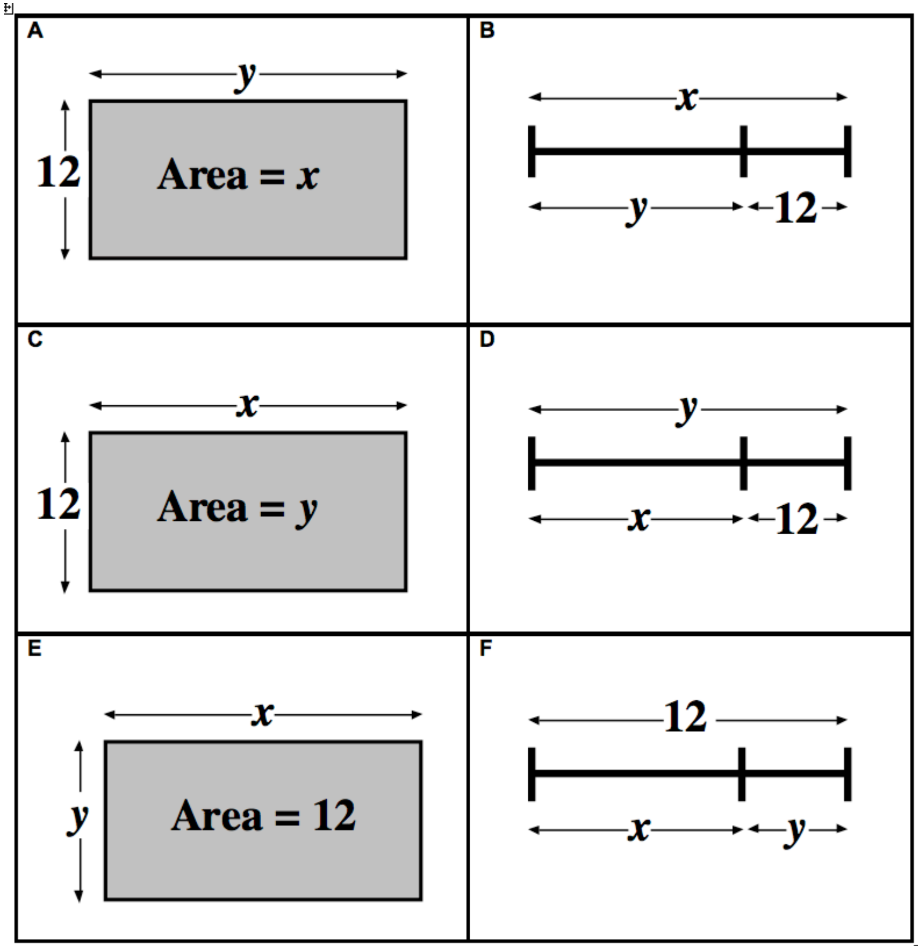
A1 $y = 5x - 10$	A2 $y = \frac{3x}{4}$
A3 $y = 40x + 60$	A4 $y = -x + 100$
A5 $y = \frac{200}{x}$	A6 $y = \frac{5}{4}\sqrt[3]{x}$
A7 $y = 10\sqrt{(x-3)^2 + 7}$	A8 $y = \frac{1}{4}x^2$
A9 $y = 30x - 5x^2$	A10 $y = 30 + 30\sin(18x)$
A11 $y = 20 + 80 \times (0.27)^x$	A12 $y = \frac{2^x}{1000}$

Multiple representations: Order of operations

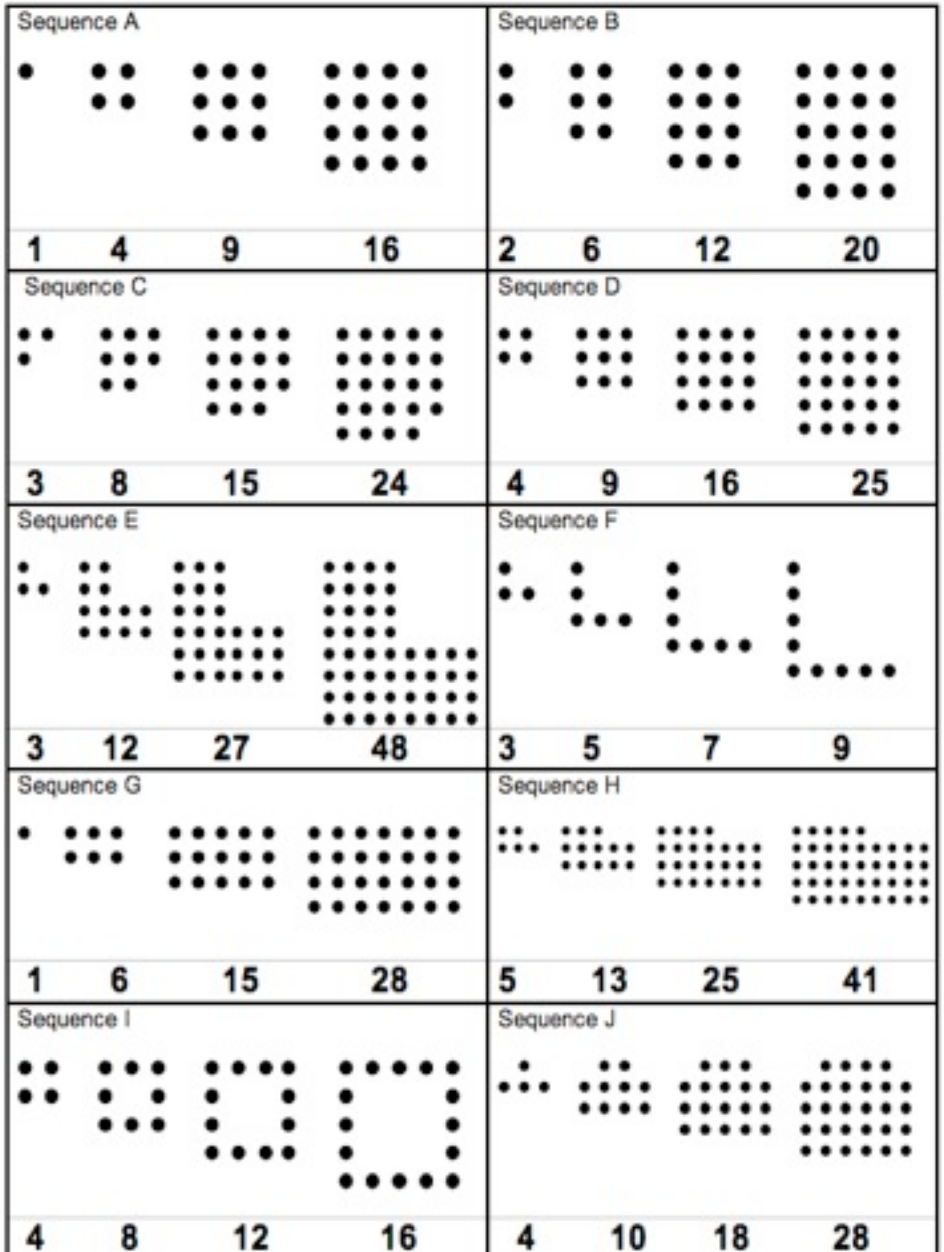


A $2\times(4+3)$	B $2\times4+3$
C $\frac{1}{2}(4+3)$	D $\frac{4+3}{2}$
E $2\times4+2\times3$	F 4×3^2
G $(4\times3)^2$	H $(4+3)^2$
I 4^2+3^2	J $4^2+2\times3\times4+3^2$
K $\frac{4}{2}+\frac{3}{2}$	L $\frac{1}{2}\times4+3$
M $4^2\times3^2$	N $3+4\times2$

Multiple representations: Interpreting equations



A $y = \frac{12}{x}$	B $y = x + 12$
C $y = x - 12$	D $y = 12 - x$
E $y = 12x$	F $y = \frac{x}{12}$
G $x = 12 - y$	H $x = \frac{y}{12}$
I $x = y + 12$	K $x = y - 12$
L $x = 12y$	M $x = \frac{12}{y}$



n^2	$4n$
n^2+2n+1	n^2+2n
n^2+n	$2n+1$
$2n^2-n$	$3n^2$
n^2+3n	$(n+1)^2$
$n(n+1)$	$n(2n-1)$
$(n+1)^2-1$	$(n+1)^2-n^2$
$(2n)^2-n^2$	$(n+1)^2-(n-1)^2$
$n+n(n+2)$	$n^2+(n+1)^2$
$(n+1)(2n+1)-n$	$n+n(n-1)$

Task "genres" that generate discussion

Classifying, naming and defining objects

- what is the same and what is different?

Interpreting multiple representations

- what is another way of showing this?

Analyzing and testing generalizations

- "always, sometimes or never true?"

Exploring structure and connections

- what happens if I change this?
- How will it affect this?

Always, sometimes or never true?

The diagonals of a quadrilateral divide the quadrilateral into equal areas.

A pentagon has fewer right angles than a rectangle.

When you cut a piece off a shape you reduce its area and perimeter.

If you double the radius of a circle, its radius doubles.

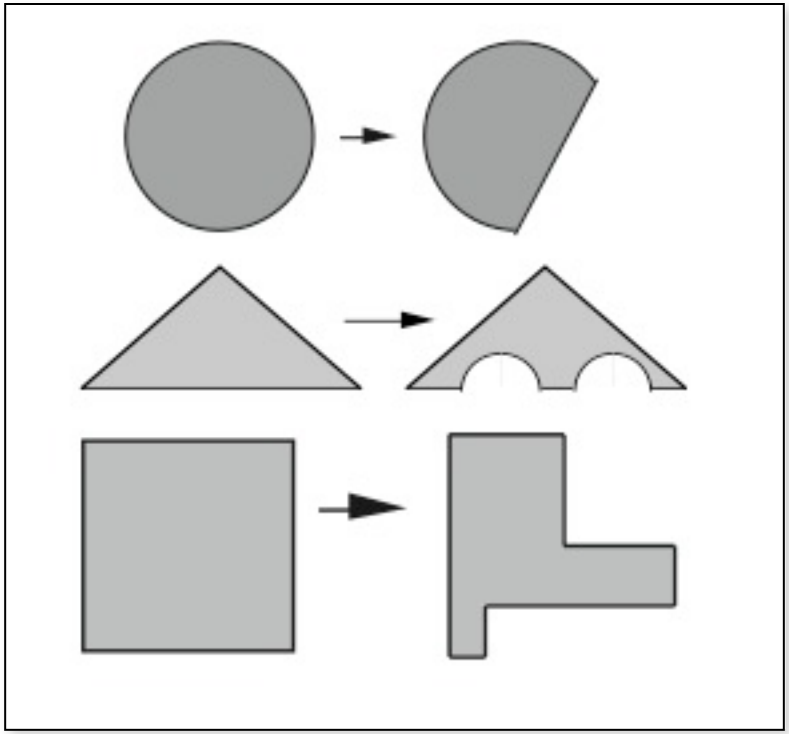
If a square and a rectangle have the same perimeter, the square will have the smaller area.

If two sides and two angles in triangle A have the same magnitude as two sides and two angles in triangle B, the triangles are congruent.

Quadrilaterals tessellate

Always, sometimes or never true?

When you cut a piece off a shape you reduce its area and perimeter



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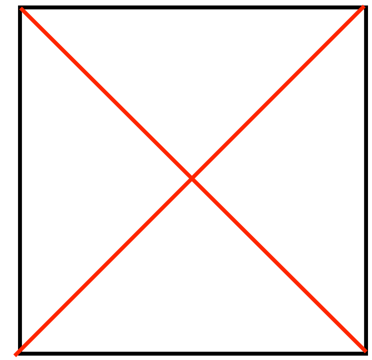
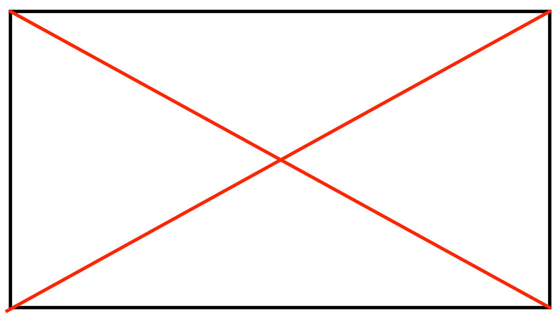
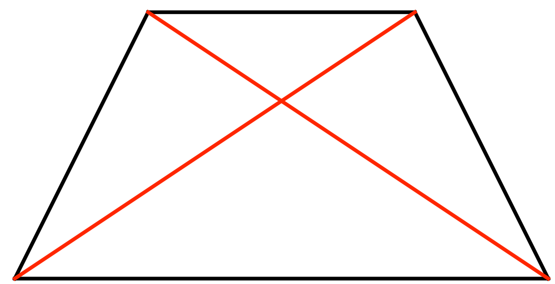
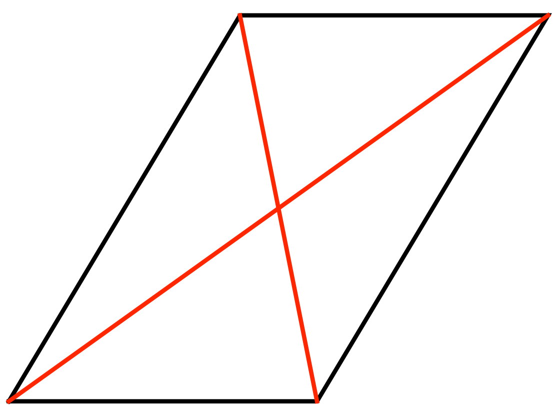
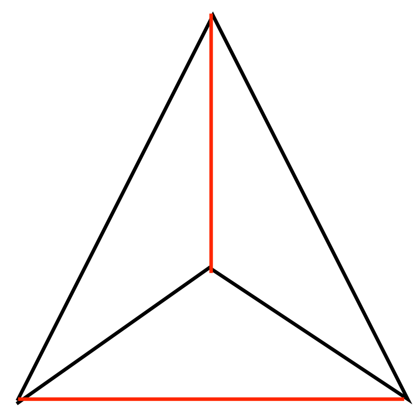




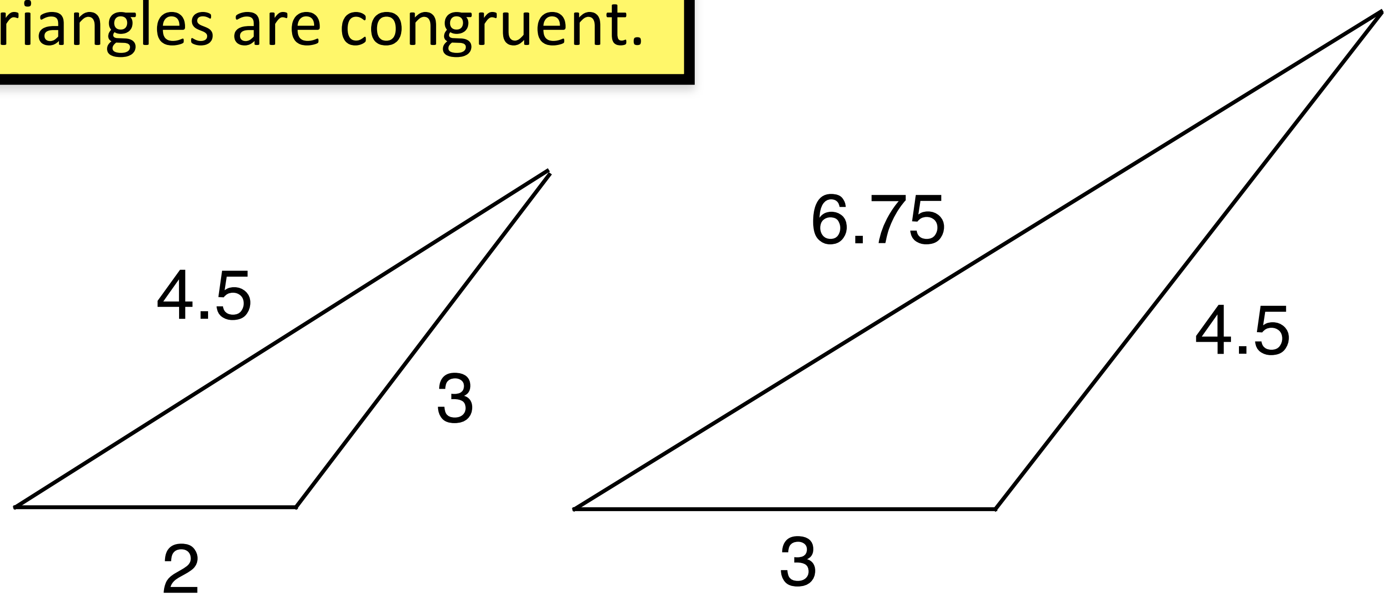






Always, sometimes or never true?

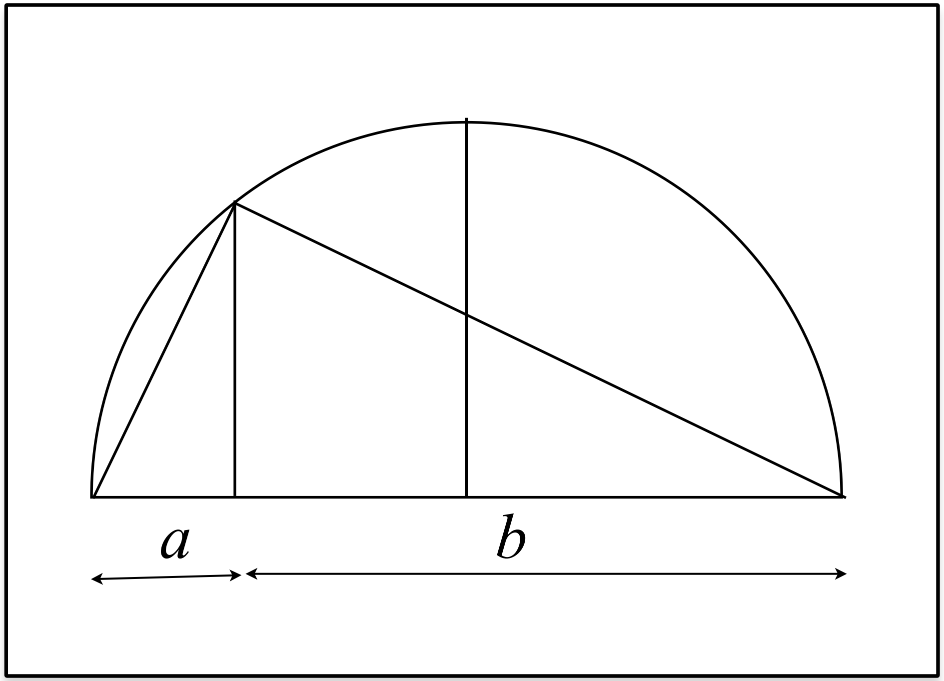
If two sides and two angles in triangle A have the same magnitude as two sides and two angles in triangle B, the triangles are congruent.



The condition has a connection with the golden ratio!

Always, sometimes or never true?

$$\frac{a+b}{2} \geq \sqrt{ab}$$



Task “genres” that generate discussion

Classifying, naming and defining objects

- what is the same and what is different?

Interpreting multiple representations

- what is another way of showing this?

Analyzing and testing generalizations

- “always, sometimes or never true?”

Exploring structure and connections

- what happens if I change this?
- How will it affect this?

"Standard" question

Van hire

Sanjay wants to hire a van to move some furniture.
He obtains the following information from two hire companies.

Bujit's Van Hire

£30 for the first 50 miles

Every mile after that costs an extra 20p

Hurt's vans

You only pay for the miles you travel.

Miles travelled	50	100	150	200
Hire charge	£16	£32	£48	£64

1. How much do Hurt's vans cost per mile?
2. Sanjay expects to travel 175 miles.
   Which company has the lower charge for this distance?
   You must show all your working.

A template for a new question

Cath wants to hire a car for a weekend.

She obtains the following information from two hire companies.

.... Car Hire

£ ....for the first

..............miles.

Every mile after that costs an

extra .... p.

.... Car Hire

Miles travelled				
Hire charge				

. . . .

. . . .

. . . .

Making and selling greeting cards

Jane wants to make exclusive hand made gift cards for charity. The cost of a kit for making the cards is £50. With this kit she can make 60 cards. She thinks they might sell at £4 each. What will be her profit if all the cards are sold?

Making and selling greeting cards

		k		
The cost of buying one kit	£	50		
		n		
The number of cards that can be made with the kit		60	cards	
		s		
The price at which each card is sold	£	4		
		p		
Total profit made if all cards are sold.	£	190		

$$p = 60 \times 4 - 50$$

$$p = ns - k$$

		k		
The cost of buying one kit	£	50		
		n		
The number of cards that can be made with the kit		60	cards	
		s		
The price at which each card is sold	£	4		
		p		
Total profit made if all cards are sold.	£			

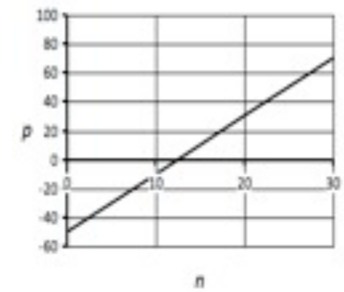
$$s = \frac{190 + 50}{60}$$

$$s = \frac{p + k}{n}$$

The cost of buying one kit	£	*k* 50	
The number of cards that can be made with the kit		*n* 60	cards
The price at which each card is sold	£	*s*	
Total profit made if all cards are sold.	£	*p* 190	

$$p = 4n - 50$$

n	0	10	20	30	40	50
p	-50	-10	30	70	110	150



		k	
The cost of buying one kit	£	**50**	
		n	
The number of cards that can be made with the kit			cards
		s	
The price at which each card is sold	£	**4**	
		p	
Total profit made if all cards are sold.	£		

$$p = ns - k \qquad s = \frac{p+k}{n} \qquad n = \frac{p+k}{s} \qquad k = ns - p$$

		k	
The cost of buying one kit	**£**		
		n	
The number of cards that can be made with the kit			**cards**
		s	
The price at which each card is sold	**£**		
		p	
Total profit made if all cards are sold.	**£**		

1: Formative Assessment

Print materials:

PD Module Guide — PDF DOC
Teacher Handouts — PDF DOC

How can I respond to students in ways that improve their learning?

The effective use of formative assessment lessons depends on the quality of feedback given by teachers to students. One important way of moving students' thinking forward is to prompt them to reconsider their reasoning by asking carefully chosen questions.

This unit contains a selection of professional activities that are designed to help teachers to reflect on:

- characteristics of their questioning that encourage students to reflect, think and reason;
- ways in which teachers might encourage students to provide extended, thoughtful answers, without being afraid of making mistakes;
- the value of showing students what reasoning means by 'thinking aloud'.

The activities described in this module are given here as a 'menu' of suggestions to help the provider select and plan. They are presented in a logical order, building up knowledge and expertise.

Any planned professional development program should offer opportunities for teachers to try new pedagogies in the classroom and then report back and reflect on their experiences. Activity D is therefore essential in the program.

About the MAP PD Modules

These modules have been developed by the Shell Centre team at the Centre for Research in Mathematics Education, University of Nottingham. They draw on successful materials developed by the team for *Bowland Maths* and *Improving Learning in Mathematics*.

Getting started

Download the print materials (links on the left) and read the main Module Guide.

Use the tabs at the top of the screen to browse the software and video which accompanies this module. (Requires JavaScript enabled and Adobe Flash Player).

Primas

Promoting Inquiry In **Mathematics** And **Science Education** Across Europe

INTERFACE: English

Professional development Dissemination Policy For parents

Professional development modules for inquiry-based, collaborative learning

Author: The University of Nottingham

These Primas professional development modules explore the pedagogical challenges that arise when introducing investigative, non-routine problem solving activities to the classroom.

The modules
Credits
Commentary

The modules are activity-based; built around a collection of example classroom activities. The intention is that, as part of the CPD process, teachers will plan inquiry-based lessons to use with their own class and, at a later meeting, report back on their experiences.

Each module includes a CPD session guide and handouts for teachers, as well as sample classroom materials and suggested lesson plans. Several of the lessons include the use of simple computer software.

Also included are several video sequences showing teachers trying these materials with their own

◂ Back to query

Credits

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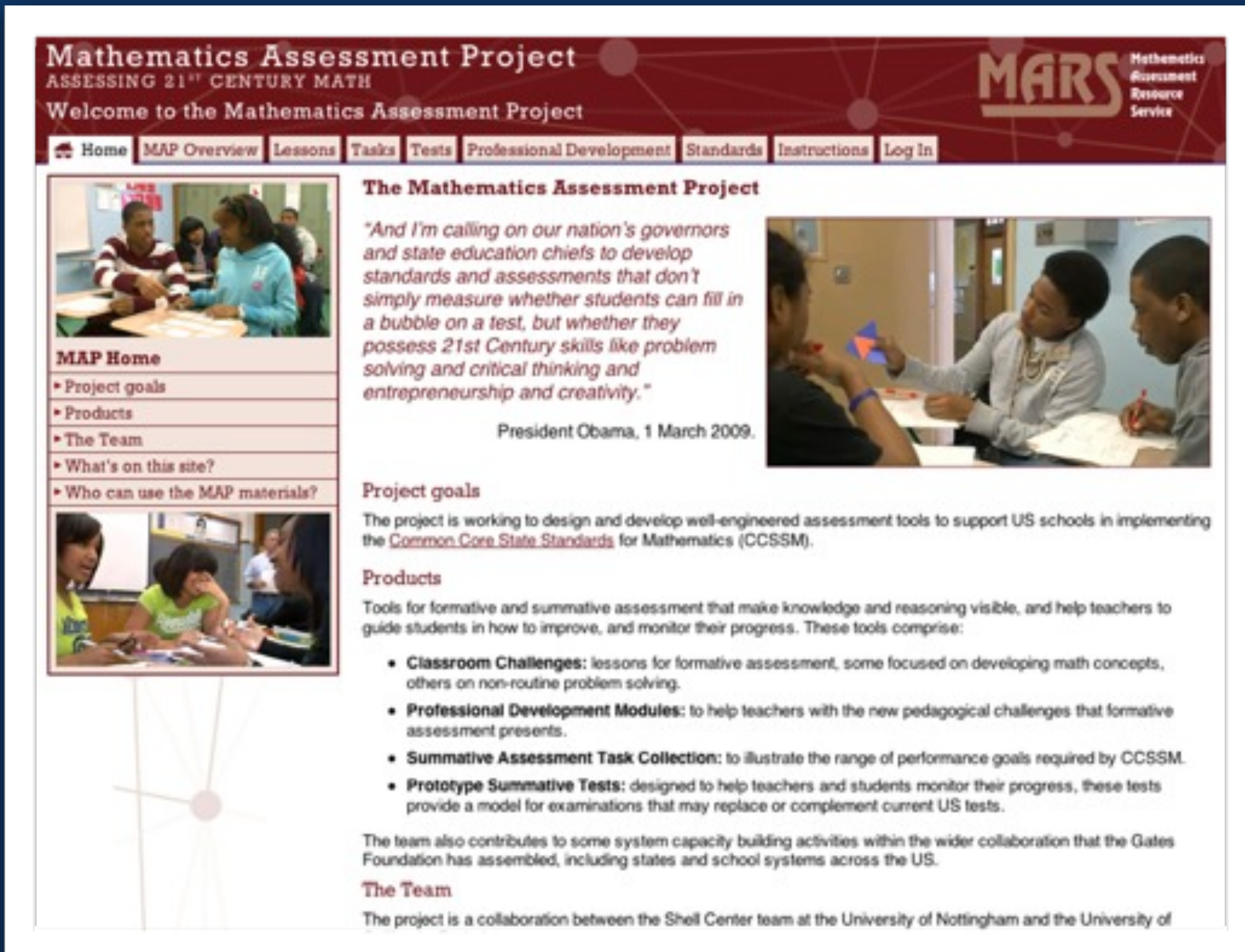
www.nottingham.ac.uk

Available languages

- Deutsch
- **English**
- Español
- Français
- Magyar
- Nederlands
- Norsk
- Slovak

http://map.mathshell.org/materials/



Mathematics Assessment Project
ASSESSING 21ST CENTURY MATH
Welcome to the Mathematics Assessment Project

MARS Mathematics Assessment Resource Service

Home | MAP Overview | Lessons | Tasks | Tests | Professional Development | Standards | Instructions | Log In

MAP Home
- Project goals
- Products
- The Team
- What's on this site?
- Who can use the MAP materials?

The Mathematics Assessment Project

"And I'm calling on our nation's governors and state education chiefs to develop standards and assessments that don't simply measure whether students can fill in a bubble on a test, but whether they possess 21st Century skills like problem solving and critical thinking and entrepreneurship and creativity."

President Obama, 1 March 2009.

Project goals

The project is working to design and develop well-engineered assessment tools to support US schools in implementing the Common Core State Standards for Mathematics (CCSSM).

Products

Tools for formative and summative assessment that make knowledge and reasoning visible, and help teachers to guide students in how to improve, and monitor their progress. These tools comprise:

- **Classroom Challenges:** lessons for formative assessment, some focused on developing math concepts, others on non-routine problem solving.
- **Professional Development Modules:** to help teachers with the new pedagogical challenges that formative assessment presents.
- **Summative Assessment Task Collection:** to illustrate the range of performance goals required by CCSSM.
- **Prototype Summative Tests:** designed to help teachers and students monitor their progress, these tests provide a model for examinations that may replace or complement current US tests.

The team also contributes to some system capacity building activities within the wider collaboration that the Gates Foundation has assembled, including states and school systems across the US.

The Team

The project is a collaboration between the Shell Center team at the University of Nottingham and the University of

The University of Nottingham

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For further details go to

http://www.nottingham.ac.uk/education/research/crme/index.aspx